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TWO RESULTS IN LEŚNIEWSKI'S MEREOLOGY

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In section 1 we prove that a certain characterization of class can be proved without the aid of auxiliary definitions. In section 2 we show that the main results in [1] still hold in the weakened system constructed by replacing the original definition of class by the characterization given in section 1.[1] In what follows we assume that the reader is acquainted with the Ontological Preliminaries in [1].

1. In the proofs given in [3] of

$$[Aa] \therefore A \,\varepsilon\, \mathbf{Kl}(a) \,.\!\equiv\!: A \,\varepsilon\, A : [B] : a \subset \mathbf{el}(B) \,.\!\equiv\!.\, A \,\varepsilon\, \mathbf{el}(B)^2$$

and

$$[AB] \therefore A \,\varepsilon\, \mathbf{el}(B) \,.\!\equiv\!: A \,\varepsilon\, A : [D] : D \,\varepsilon\, \mathbf{el}(A) \,.\!\supset\!.\, [\exists F] \,.\, F \,\varepsilon\, \mathbf{el}(D) \,.\, F \,\varepsilon\, \mathbf{el}(B)$$

Leśniewski's definition of set plays an important role. Sobociński has asked whether this definition is creative with respect to the above two theorems, or, if not, whether some auxiliary definition is required. The answer is no. The proof follows.

An axiom system for mereology, denoted $\mathcal{M}$, is given by *A1* through *A6* with *D1*. (This is not an independent axiom set. (*A1* and *A2* are derivable from the rest).)

A1 $[A] : A \,\varepsilon\, A \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{el}(A)$

A2 $[AB] : A \,\varepsilon\, \mathbf{el}(B) \,.\, B \,\varepsilon\, \mathbf{el}(A) \,.\!\supset\!.\, A = B$

A3 $[ABC] : A \,\varepsilon\, \mathbf{el}(B) \,.\, B \,\varepsilon\, \mathbf{el}(C) \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{el}(C)$

A4 $[AB] : A \,\varepsilon\, \mathbf{el}(B) \,.\!\supset\!.\, B \,\varepsilon\, B$

D1 $[Aa] \therefore A \,\varepsilon\, \mathbf{Kl}(a) \,.\!\equiv\!: A \,\varepsilon\, A : [D] : D \,\varepsilon\, a \,.\!\supset\!.\, D \,\varepsilon\, \mathbf{el}(A) : [D] : D \,\varepsilon\, \mathbf{el}(A) \,.\!\supset\!.\, [\exists EF] \,.$
$E \,\varepsilon\, a \,.\, F \,\varepsilon\, \mathbf{el}(D) \,.\, F \,\varepsilon\, \mathbf{el}(E)$

A5 $[ABa] : A \,\varepsilon\, \mathbf{Kl}(a) \,.\, B \,\varepsilon\, \mathbf{Kl}(a) \,.\!\supset\!.\, A = B$

A6 $[Aa] : A \,\varepsilon\, a \,.\!\supset\!.\, [\exists B] \,.\, B \,\varepsilon\, \mathbf{Kl}(a)$

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1. These two results have been included in the same paper because, though they are very different, their proofs are related.

2. The theorem in [3] actually used $\sqsubset$; Lejewski remarked that $\sqsubset$ could be replaced by $\subset$.

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