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A STRONGER THEOREM CONCERNING THE NON-EXISTENCE OF COMBINATORIAL DESIGNS ON INFINITE SETS

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The aim of the present paper is to show that certain combinatorial designs on infinite sets cannot exist. To be precise we introduce the following terminology. If M is a set and p a cardinal number $\leq \overline{\overline{M}}$ (the cardinal number of M) then $[M]^p$ is the collection of all subsets of M having cardinality p.

Definition 1. A family F is called a *p-tuple family* of M if and only if (i) $F \subset [M]^p$ and (ii) $x, y \in F$ and $x \subset y$ implies $x = y$.

Definition 2. Let F and G be two families of subsets of M. G is called a *Steiner cover* of F if and only if for every $x \in F$ there is exactly one $y \in G$ such that $x \subset y$.

It is now possible to state the main result of the present paper.

Theorem 3. *Let α, β and γ be ordinal numbers such that*

(i) $\alpha < \beta < \gamma$
(ii) $\mathrm{cf}(\omega_\gamma) \leq \omega_\alpha$
(iii) $\aleph_\beta^{\aleph_\alpha} \leq \aleph_\gamma$.

Then, in every set M of cardinality $\aleph_\gamma$ there exists an $\aleph_\alpha$-tuple family F of M such that there does not exist a family $G \subset [M]^{\aleph_\beta}$ which is a Steiner cover of F.

N.B. It should be noted that this result subsumes the main result of [1] (denoted there as Theorem 6) and [2] (denoted as Theorem 4) as special cases. It should also be noted that the proofs offered for both of these results contain errors. This fact was kindly pointed out to me by Professor E. C. Milner of the University of Calgary, Alberta, Canada, whom the present author wishes to take this opportunity to thank.

We begin with some preliminaries.

Definition 4. Let F and G be families of subsets of M and n a non-zero cardinal number. G is called an *n-spoiler* of F if and only if for every $x \in F$ and every $y \in [M]^n$ there is a $z \in G$ such that $z \subset x \cup y$.

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