Notre Dame Journal of Formal Logic
Volume XIV, Number 4, October 1973
NDJFAM

CONCERNING THE QUANTIFIER ALGEBRAS IN THE SENSE OF PINTER

BOLESŁAW SOBOCIŃSKI

1 In [3],[1] *cf.* especially p. 362, section **2.1** and p. 365, section **4.1**, C. C. Pinter formulated and investigated an algebraic system which he called the quantifier algebras and which, in conformity with the style used in [6], pp. 529-530, [5], p. 111, section **1**, and [4], is defined here as follows:

(**A**) *Any algebraic structure*

$$\mathfrak{A} = \langle A, +, \times, -, 0, 1, \mathbf{S}^{\kappa}_{\lambda}, \exists_{\kappa}\rangle_{\kappa,\lambda<\alpha}$$

where α *is any ordinal number,* $+$ *and* $\times$ *are two binary operations, and* $-$, $\mathbf{S}^{\kappa}_{\lambda}$ *(for any ordinal numbers* $\kappa, \lambda < \alpha$*) and* $\exists_{\kappa}$ *(for any ordinal number* $\kappa < \alpha$*) are the unary operations defined on the carrier set* A*, and* 0 *and* 1 *are two constant elements belonging to* A*, is a quantifier algebra of dimension* α*, if it satisfies the following postulates:*

C0	*the structure* $\langle A, +, \times, -, 0, 1\rangle$ *is a Boolean algebra*
Q1	$[a\kappa\lambda]: a \in A . \kappa, \lambda < \alpha . \supset . \mathbf{S}^{\kappa}_{\lambda}(-a) = -\mathbf{S}^{\kappa}_{\lambda}a$
Q2	$[ab\kappa\lambda]: a, b \in A . \kappa, \lambda < \alpha . \supset . \mathbf{S}^{\kappa}_{\lambda}(a + b) = \mathbf{S}^{\kappa}_{\lambda}a + \mathbf{S}^{\kappa}_{\lambda}b$
Q3	$[a\kappa]: a \in A . \kappa < \alpha . \supset . \mathbf{S}^{\kappa}_{\kappa}a = a$
Q4	$[a\kappa\lambda\mu]: a \in A . \kappa, \lambda, \mu < \alpha . \supset . \mathbf{S}^{\kappa}_{\lambda}\mathbf{S}^{\mu}_{\kappa}a = \mathbf{S}^{\kappa}_{\lambda}\mathbf{S}^{\mu}_{\lambda}a$
Q5	$[ab\kappa]: a, b \in A . \kappa < \alpha . \supset . \exists_{\kappa}(a + b) = \exists_{\kappa}a + \exists_{\kappa}b$
Q6	$[a\kappa]: a \in A . \kappa < \alpha . \supset . a \leq \exists_{\kappa}a$
Q7	$[a\kappa\lambda]: a \in A . \kappa, \lambda < \alpha . \supset . \mathbf{S}^{\kappa}_{\lambda}\exists_{\kappa}a = \exists_{\kappa}a$
Q8	$[a\kappa\lambda]: a \in A . \kappa, \lambda < \alpha . \kappa \neq \lambda . \supset . \exists_{\kappa}\mathbf{S}^{\kappa}_{\lambda}a = \mathbf{S}^{\kappa}_{\lambda}a$
Q9	$[a\kappa\lambda\mu]: a \in A . \kappa, \lambda, \mu < \alpha . \mu \neq \kappa, \lambda . \supset . \mathbf{S}^{\kappa}_{\lambda}\exists_{\mu}a = \exists_{\mu}\mathbf{S}^{\kappa}_{\lambda}a$

Moreover, if, besides 0 *and* 1, *the carrier set* A *of the structure* $\mathfrak{A}$ *contains also the constant elements* $\mathbf{e}_{\kappa\lambda}$ *(for any ordinal numbers* $\kappa, \lambda < \alpha$*) such that* $\mathfrak{A}$ *satisfies the following two additional postulates:*

1. An acquaintance with papers [3], [6] and [4] and with the symbolism used in [6] is presupposed.

Received September 2, 1972