Notre Dame Journal of Formal Logic
Volume XIV, Number 3, July 1973
NDJFAM

NOTE ABOUT THE BOOLEAN PARTS OF THE EXTENDED BOOLEAN ALGEBRAS

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Throughout this note[1] the Boolean algebras extended by the additional extra-Boolean operations and postulates and containing the so-called Boolean part, in short **BA**, i.e., a postulate

C0 *the structure* $\langle A, +, \times, -, 0, 1\rangle$ *is a Boolean algebra*

will be called the extended Boolean algebras. In [3] and [2] it has been proved that in several systems of the extended Boolean algebras the postulate *C0* can be substituted for the postulates weaker than *C0*, namely either by

*C0** *the structure* $\langle A, +, \times, -, 0, 1\rangle$ *is a non-associative Newman algebra*

or by

*C0*** *the structure* $\langle A, +, \times, -, 0, 1\rangle$ *is a dual non-associative Newman algebra.*

1 An inspection of the deductions presented in [3] and [2] suggests the following elementary, but general lemma:

Lemma I. *Let* $\mathfrak{M}$ *be an arbitrary extended Boolean algebra,* M *be the carrier set of* $\mathfrak{M}$*,* $\mathcal{A}$ *be the set of all primitive extra-Boolean operations occurring in the definition of* $\mathfrak{M}$*, and* $\mathcal{B}$ *be the set of all extra-Boolean postulates accepted in* $\mathfrak{M}$*. Let* Z *be a unary extra-Boolean operation which either belongs to* $\mathcal{A}$ *or is definable in the field of the postulates of* $\mathfrak{M}$*. Then:*

(i) *if* Z *either belongs to* $\mathcal{A}$ *or is syntactically definable in the field of C0*, extended by the postulates belonging to* $\mathcal{B}$*, and in that field a formula*

A1 $[a]:a \in M.\supset.\, a + \mathrm{Z}a = \mathrm{Z}a$

1. An acquaintance with [3] and [2] is presupposed.

Received November 7, 1971