

A NEW AXIOMATIZATION OF MODAL SYSTEM K1.2

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In [5], p. 316, system K1.2 is defined as an extension of S4 obtained by the addition of the axiom

H1 $\mathcal{C}pLCMpp$

to that system, and in [4], pp. 352-355, section 2.5, it has been proved that in the field of S4 the axiom H1 is inferentially equivalent to its consequence

H2 $\mathcal{C}LMpCpLp$.

In [2], p. 396, Goldblatt has shown that the same equivalence holds in the field of system S2. In this note it will be proved that the addition either of H1 or of H2, as a new axiom, to S2 generates system K1.2.

Proof: Since it is self-evident that in the field of S2 H1 implies H2, let us assume system S2 (classically formalized) and formula H2. Then:

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|-------|---|--|
| $Z1$ | $\mathcal{C}Lpp$ | [S1] |
| $Z2$ | $\mathcal{C}LCpqCLpLq$ | [S1°] |
| $Z3$ | $\mathcal{C}\mathcal{C}Cpqr\mathcal{C}qr$ | [S2°, BR (Becker Rule), cf. [1], p. 73, 46.1 and 46.2] |
| $Z4$ | $\mathcal{C}\mathcal{C}pCpq\mathcal{C}pq$ | [S2°, BR] |
| $Z5$ | $\mathcal{C}CMpLqLCpq$ | [S1, cf. [3], p. 156] |
| $Z6$ | $\mathcal{C}CLpMqMCPq$ | [S2°, cf. [6], pp. 71-72, Lemma] |
| $Z7$ | $\mathcal{C}MCMpLqMLCPq$ | [Z5; S2°, BR] |
| $Z8$ | $\mathcal{C}LMCMpLqLMLCPq$ | [Z7; S2°, BR] |
| $Z9$ | $\mathcal{C}LCLpMqLMCPq$ | [Z6; S2°, BR] |
| $Z10$ | $\mathcal{C}MLCLpMqMLMCPq$ | [Z9; S2°, BR] |
| $Z11$ | $\mathcal{C}LMLCLpMqLMLMCPq$ | [Z10; S2°, BR] |
| $Z12$ | $LMCMpp$ | [Z9, p/Mp, q/p; Z1, p/Mp] |
| $Z13$ | $LMLCLpp$ | [Z8, p/Lp, q/p; Z12, p/Lp] |
| $Z14$ | $LMLMCMpp$ | [Z11, p/Mp, q/p; Z13, p/Mp] |
| $Z15$ | $CLMCMppLLMCMpp$ | [H2, p/LMCMpp; Z14] |
| $Z16$ | $LLMCMpp$ | [Z15; Z12; S1°, cf. [1], p. 53, 32.221] |
| $Z17$ | $LLMCpLp$ | [Z16, p/Np; S1°] |
| $Z18$ | $\mathcal{C}LLMp\mathcal{C}pLp$ | [H2; S2°, BR] |