Notre Dame Journal of Formal Logic
Volume XIV, Number 3, July 1973
NDJFAM

CONCERNING THE PROPER AXIOM FOR S4.04 AND SOME RELATED SYSTEMS

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This paper examines the group of modal axioms covered by the general schema

(X) $$Xp \rightarrow (p \rightarrow Lp)$$

where X is an affirmative modality of S4. Familiarity is assumed with the properties of maximal-consistent sets of wff, and with the post-Henkin method of completeness proofs. Soundness proofs are left to the reader throughout.

(X) yields seven cases:

Case 1. Zeman's S4.04 axiom

L1 $LMLp \rightarrow (p \rightarrow Lp)$ *cf*. [5], p. 250

In the field of S4, **L1** is equivalent to

L2 $p \rightarrow L(MLp \rightarrow p)$

That **L1** is a consequence of **L2** is easy to see. For the converse we have[1]

(1)	$MLp \rightarrow ML(MLp \rightarrow p)$	C2
(2)	$\sim MLp \rightarrow (MLp \rightarrow p)$	PC
(3)	$\sim LMMLp \rightarrow ML(MLp \rightarrow p)$	(2), C2
(4)	$\sim MLp \rightarrow ML(MLp \rightarrow p)$	(3), S4, PC
(5)	$ML(MLp \rightarrow p)$	(1), (4), PC
(6)	$LML(MLp \rightarrow p)$	(5), T^0
(7)	$LML(MLp \rightarrow p) \rightarrow ((MLp \rightarrow p) \rightarrow L(MLp \rightarrow p))$	L1, $p/MLp \rightarrow p$
(8)	$(MLp \rightarrow p) \rightarrow L(MLp \rightarrow p)$	(6), (7), PC
(9)	$p \rightarrow L(MLp \rightarrow p)$	(8), PC

We now present a semantic analysis that distinguishes **L1** and **L2** in

1. This proof is due to Professor G. E. Hughes.

Received November 19, 1971