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A DEDUCTION THEOREM FOR RESTRICTED GENERALITY

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In this paper, a deduction theorem for restricted generality (Ξ) will be proved on the basis of a finite number of axioms which do not contain variables. The theorem is in such a form as to avoid both Curry's paradox[1] and the Kleene Rosser paradox.[2] In fact it can be shown that nothing inconsistent can be proved using this form of the deduction theorem and the basic rules given below.[3]

An iterated form of the theorem can also be derived, as well as deduction theorems for P (implication) and Π (universal generality).

1. *The combinatory system* The notation we use in this paper is as in [4], in addition we take $\mathsf{H}x$ to stand for "x is a proposition." The system in which we prove the deduction theorem will contain at least two rules, others are expressible in terms of them. The first is the basic rule for restricted generality Ξ:

Rule Ξ. $\Xi xy, xu \vdash yu$.

Note that xu may be interpreted as "u has the property x" or as "u is an element of the class x" and Ξxy may be interpreted as "for all u for which xu holds, yu also holds" or as "x is a subclass of y." Ξxy will also be written as $xu \supset_u yu$. The second rule is one for equality (Q).

Rule Eq. $\mathsf{Q}xy, x \vdash y$.

The system may also include any set of axioms without variables. These can include axioms for equality such as $\vdash \mathsf{Q}XX$, for every primitive ob X, (there are a finite number of these: Ξ, Q, K, S so far) and

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1. See [4] Chapter 5. It is obtained when implication P is defined by $\mathsf{P} \equiv [x,y]\ \Xi(\mathsf{K}x)(\mathsf{K}y)$.

2. See Kleene and Rosser [5]. This form of the theorem also avoids a generalized version of the Kleene Rosser paradox. This version of the paradox will appear in a later paper.

3. This is done in an as yet unpublished paper by H. B. Curry and the author.

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