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ON AN ALLEGED CONTRADICTION LURKING IN FREGE'S *BEGRIFFSSCHRIFT*

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Jean van Heijenoort, in his introduction [1] to Bauer-Mengelberg's translation of Frege's *Begriffsschrift* [2], claims to see a contradiction lurking in the logical system of that work:*

> Frege allows a functional letter to occur in a quantifier. . . . The result is that the difference between function and argument is blurred . . . in the derivation of formula (77) he substitutes $\mathfrak{F}$ [a quantificationally bound function letter] for $\mathfrak{a}$ [a quantificationally bound individual variable] in $f(\mathfrak{a})$, at least as an intermediate step. If we also observe that in the derivation of formula (91) he substitutes $\mathfrak{F}$ for f [a "free" function letter], we see that he is on the brink of a paradox. He will fall into the abyss when (1891) he introduces the course-of-values of a function as something "complete in itself," which may be taken as an argument.[1]

Van Heijenoort is mistaken in supposing that any paradox can arise from the derivations he cites in the *Begriffsschrift*. In that early work, Frege is pioneering the development of quantificational logic. While he does not yet have all the machinery or the terminology to precisely spell out the distinction between what he would later call "first-level" and "second-level" functions, he never confuses the two. And because his functions occur in "levels," Frege's functional calculus (including that in the *Begriffsschrift* of 1879) is free of the kind of paradox which, beginning in 1891,[2] does afflict his set theory. Frege himself points this out in a letter to Russell in June of 1902 [4] when responding to Russell's letter [5] about the discovery of a paradox.

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1. See [1], p. 3.

2. In that year, in [3] Frege first introduced the notion of the "course-of-values" {Wertverlauf} of a function.

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