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MANY-VALUED LOGICS AND THE LEWIS PARADOXES

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W. C. Wilcox has recently suggested that certain features of C. I. Lewis's paradoxes of material implication can be used to "lend some justification" to Łukasiewicz's well known claim that there is no interesting n-valued logic between 3-valued and infinitely many-valued systems.[1] The purpose of this note is to show just the contrary, that Lewis's paradoxes can in fact be used to lend some doubt to Łukasiewicz's claim.

One way in which Lewis tries to establish the paradoxical character of material implication and equivalence is by listing, and then formulating in language, some peculiar sounding theorems of *Principia Mathematica*:[2]

$p \supset (q \supset p)$	$\sim(p \supset q) \supset p$	$pq \supset (p \equiv q)$
$\sim p \supset (p \supset q)$	$\sim(p \supset q) \supset \sim q$	$\sim p \sim q \supset (p \equiv q)$
$pq \supset (p \supset q)$	$\sim(p \supset q) \supset p \sim q$	$p \sim q \supset \sim(p \equiv q)$
$pq \supset (q \supset p)$	$\sim(p \supset q) \supset (p \supset \sim q)$	etc.
$\sim p \sim q \supset (p \supset q)$	$\sim(p \supset q) \supset (\sim p \supset q)$	
$\sim p \sim q \supset (q \supset p)$	$\sim(p \supset q) \supset (\sim p \supset \sim q)$	

But a more dramatic, if not more effective, technique used by Lewis against the horseshoe requires that we imagine a large number of true statements and an equal number of false statements—the more variegated the better—written on slips of paper and randomized in a large hat. Then let two of these statements be drawn at random. The probabilities of the

1. "On infinite matrices and the paradoxes of material implication," *Notre Dame Journal of Formal Logic*, vol. XI (1970), p. 254. It should be remarked that Łukasiewicz claims only that no "philosophical significance" attaches to n-valued ($3 < n < \infty$) logics: "Philosophical remarks on many-valued systems of propositional logic," *Polish Logic*, Storrs McCall, Ed., Oxford (1967), p. 60.

2. Lewis and Langford, *Symbolic Logic*, Century Co., New York (1932), esp. pp. 85-89. It is interesting that most, if not all, the ten "illogical" inferences rejected by William S. Cooper ("Propositional Logic of Ordinary Discourse," *Inquiry* (1968), pp. 295-320) appear to be variations on Lewis's list.

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