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SIMULTANEOUS *VERSUS* SUCCESSIVE QUANTIFICATION

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In standard predicate calculus, if u and v are distinct variables, then "for all u and v, Puv" is satisfactorily restated as "for all u, for all v, Puv"; symbolically:

$$\forall(u, v)Puv \text{ as } \forall u\forall vPuv \ .$$

Similarly, "there are u and v such that Puv" is satisfactorily restated as "there is a u such that there is a v such that Puv"; symbolically:

$$\exists(u, v)Puv \text{ as } \exists u\exists vPuv \ .$$

On the other hand, in standard predicate calculus with equality, it is *not* correct to restate "there exist unique u and v such that Puv" as "there exists a unique u such that there exists a unique v such that Puv"; symbolically:

$$\exists!(u, v)Puv \textit{ versus } \exists!u\exists!vPuv \ ,$$

where

$$\exists!vPuv \text{ abbreviates } \exists vPuv \wedge \forall v\forall v_1(Puv \wedge Puv_1 \rightarrow v = v_1),$$
$$\exists!(u, v)Puv \text{ abbreviates } \exists u\exists vPuv \wedge \forall u\forall u_1\forall v\forall v_1(Puv \wedge Pu_1v_1 \rightarrow u = u_1 \wedge v = v_1),$$

and u_1 and v_1 are distinct variables not occurring in $\exists u\exists vPuv$. We have the following counterexample. In the theory of real (or complex) numbers,

$$\exists!(x, y)\ (x = y^2)$$

is false (since there are many pairs (x, y) such that $x = y^2$), but

$$\exists!x\exists!y(x = y^2)$$

is true (since only 0 has exactly one square root). Simultaneous unique existence is often used in stating theorems, as in the following special case of the division algorithm in the theory of natural numbers:

$$\exists!(x, y)\ (0 = 1 \cdot x + y \wedge y < 1) \ .$$

It is an exercise in predicate calculus with equality to show that

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