224
Notre Dame Journal of Formal Logic
Volume XIV, Number 2, April 1973
NDJFAM

ON CONSERVING POSITIVE LOGICS

ROBERT K. MEYER

Let L^+ be a sentential logic without negation. One frequently wishes to know which classically valid negation axioms can be added *conservatively* to L^+, in the sense that the negation-free fragment of the resulting logic L is precisely L^+. The question becomes more urgent as the strength of the axioms to be added increases, for it frequently happens that one cannot add together axioms sufficient for the full classical principles of double negation, excluded middle, and contraposition conservatively.[1]

In the present paper, we shall develop a method which will enable us to prove, for several interesting systems, that their negation-free fragments are determined by their negation-free axioms. We take as the negation axioms to be added those given by Anderson and Belnap for their system E of entailment, namely

A1. $\overline{\overline{A}} \rightarrow A$
A2. $(A \rightarrow \overline{B}) \rightarrow (B \rightarrow \overline{A})$
A3. $(A \rightarrow \overline{A}) \rightarrow \overline{A}$.

We note in passing that these axioms lead in E (and in related systems) to the theoremhood of all forms of the double negation laws, the DeMorgan laws, contraposition laws, and laws of excluded middle and non-contradiction. In short they are strong axioms, raising non-trivial questions of conservative extension.[2]

1. For example, the addition of plausible axioms expressing all these principles causes the negation-free fragment J^+ of the intuitionist sentential calculus to collapse into the classical calculus K, as is well known. Cf. [7].

2. Anderson in [1] explicitly takes note of the conservative extension question for the system E^+ determined by the negation-free axioms and rules of E. He lists this as an open problem for E significant not only in its own right but on account of relations between E and J investigated in [4]; the problem is similarly interesting for the Anderson-Belnap system R, which is even more intimately related to J. Cf. [11].

Received August 28, 1970