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THE COSUBSTITUTION CONDITION

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1. *Introduction*. Let n be a natural number, $n \geq 2$, and $N = \{1, 2, . . , n\}$. Martin [3] showed in 1954 that necessary conditions for a two-place functor to be a Sheffer function are that it should possess none of the properties of proper closure, t-closure, proper substitution or cosubstitution. He proved these conditions sufficient in the 3-valued case. Foxley [1] demonstrated that, in the 3-valued case, any function which possessed the cosubstitution property must also possess at least one of the properties of proper closure, proper substitution or t-closure. We shall establish the corresponding result for n-valued logic. Initially we establish a necessary and sufficient condition for a function to be t-closing. By investigating the conditions implied by the cosubstitution property it will follow that if F possesses the cosubstitution property for a decomposition of the n truth values into less than n classes then it will also possess the proper substitution property for the same decomposition. In the remaining case of a decomposition of the n truth values into exactly n classes it will be shown that F will possess at least one of the properties of proper closure, proper substitution or t-closure if it possesses the cosubstitution property for such a decomposition.

Before proceeding any further we will introduce definitions of these terms as given by Martin [3]. Suppose we have a decomposition of the n marks into two or more disjoint, non-empty classes. If $a, b \in N$ we write $a \sim b$ to indicate that a and b are elements of the same class. Let a', b', c', d', e', f' be logical constants taking the truth values a, b, c, d, e, f respectively ($a, b, c, d, e, f \in N$). A binary functor F satisfies the *substitution law* if, for any a, b, c, d, whenever $a \sim c$ and $b \sim d$ then $e \sim f$ where $Fa'b' =_T e'$ and $Fc'd' =_T f'$. If F is a binary functor such that whenever $e \sim f$ and $Fa'b' =_T e'$, $Fc'd' =_T f'$ then either $a \sim c$ or $b \sim d$ then F satisfies the *cosubstitution law*. We say F has the *proper substitution property* if there is a decomposition of the n truth values into less than n classes for which F satisfies the substitution law. Similarly F has the *cosubstitution property* if there is a decomposition of the n truth values for which F satisfies the cosubstitution law.

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