516
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IN SO MANY POSSIBLE WORLDS

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Ordinary modal logic deals with the notion of a proposition being true in at least one possible world. This makes it natural to consider the notion of a proposition being true in k possible worlds for any nonnegative integer k. Such a notion would stand to Tarski's numerical quantifiers as ordinary possibility stands to the existential quantifier.

In this paper[1] I present several logics for numerical possibility. First I give the syntax and semantics for a minimal such logic (sections **1** and **2**); then I prove its completeness (sections **3** and **4**); and finally I show how to extend this result to other logics (section **5**).

1. *The Logic* Kn. The logic Kn is defined as follows.

Formation Rules: Formulas are constructed in the usual way from a set V of propositional variables $p_1, p_2, \ldots$, the binary operator $\vee$ (or), the unary operators - (not), L (necessity) and M_k, $k = 2, 3, \ldots$, and parentheses (and).

Throughout the paper I observe some familiar conventions: A, B, C, D and E, with or without subscripts, range over formulas; $\rightarrow$, $\leftrightarrow$, M (possibility) etc. are given standard definitions; all expressions are used autonomously; and parentheses are omitted from formulas in an obvious way. $M_0 A$ abbreviates $A \rightarrow A$, $M_1 A$ abbreviates MA and $Q_k A$ abbreviates $M_k A$ & - $M_{k+1} A$, $k = 0, 1, \ldots$. $M_k A$ is taken to mean A is true in at least k possible worlds; so $Q_k A$ means A is true in exactly k possible worlds (see section **2**). $\vdash A$ means A is a theorem of Kn.

Transformation Rules:

Axiom-schemes (where $k, l = 1, 2, \ldots$*)*

1. All tautologous formulas

1. The results of this paper are contained in my doctorate thesis, submitted to the University of Warwick in 1969. I am greatly indebted to my supervisor, the late Arthur Prior. Without his help and encouragement this paper would never have been written.

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