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THE COMPLETENESS OF S1 AND SOME RELATED SYSTEMS

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The system S1, although dating back to Lewis and Langford in 1932 [12] has proved singularly recalcitrant to the algebraic and semantic techniques applied so successfully to other modal logics. In this paper* we define S1-algebras (section **2**), use them to prove the finite model property for S1 (section **3**), introduce a semantical definition of S1-validity (section **4**) and make a few remarks about various other systems which seem amenable to the S1 treatment (section **5**).

1 *The system* S1. We use the basis for S1 given by Lemmon in [9, p. 178]. Lemmon takes $\sim$, $\supset$, and L as primitive with the definitions[1]:

Def $\prec$: $(\alpha \prec \beta) =_{df} L(\alpha \supset \beta)$
Def $=$: $(\alpha = \beta) =_{df} ((\alpha \prec \beta) \,.\, (\beta \prec \alpha))$
Def M: $M\alpha =_{df} \sim L \sim \alpha$

The axioms are:

1.1 $Lp \supset p$
1.2 $(L(p \supset q) \,.\, L(q \supset r)) \supset L(p \supset r)$

and the rules:

1.3 If α is a **PC**-tautology or an axiom then $L\alpha$ is a theorem.
1.4 Uniform substitution for propositional variables.
1.5 Modus Ponens: $\vdash \alpha$, $\vdash \alpha \supset \beta \rightarrow \vdash \beta$
1.6 Substitution of proved strict equivalents.

In view of 1.3 and 1.6 the choice of primitives is immaterial. The following strict equivalences will frequently be tacitly assumed in what follows:

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*This paper was written in 1969 before the publication of A. Shukla's work on S1 in [15]. A comparison between his algebras and ours is instructive. I am indebted to Mr. K. E. Pledger of the Victoria University of Wellington Mathematics Department for drawing my attention to some errors in an earlier draft of this paper.

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