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NATURAL DEDUCTION RULES FOR MODAL LOGICS

THOMAS W. SATRE

Natural deduction rules for systems of modal logic have been formulated in the style of Gentzen by Ohnishi, Matsumoto, Kanger, and Curry. The purpose of this paper is to formulate natural deduction rules for the S-, E-, and D-families of systems studied by Lemmon in [6]. Some of the rules for modal operators contained in this paper have been employed by others. The remainder are, to the author's knowledge, new. The style of presentation of the rules shall be that of Lemmon's natural deduction rules for propositional calculus in [5]; similar formulations of rules are to be found in [15]. In section **1** the standard proof of the deduction theorem is shown to hold in the three families of systems to be studied. In section **2** each system of these families is given a deductively equivalent formalization by means of natural deduction rules. Finally, some suggestions concerning interpretation of modal logics are offered in section **3**.

1. In [6] Lemmon showed that the Lewis systems S1-S5, together with S0.5, S0.9, and the families of systems D1-D5 and E1-E5, can be axiomatized as extensions of classical propositional calculus. The propositional calculus basis common to all these systems shall be referred to as **PC** and shall have the axioms and rules:

PC1: $p \supset (q \supset p)$
PC2: $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
PC3: $(-q \supset -p) \supset ((-q \supset p) \supset q)$
PC4: *If* $\vdash A$ *and* B *comes from* A *by uniform substitution for propositional variables of* A, *then* $\vdash B$.
PC5: *From* A *and* $A \supset B$, B *may be inferred*.

The usual definitions of &, v, and ≡ are assumed. Further, it is assumed that the reader is acquainted with the modal rules (a)-(D), the modal axioms (1)-(5), and the definitions of $\multimap$, $\diamond$, and $\equiv$ of [6].[1]

1. In the notation of [6] these are respectively C′, M, and E′. The primitive modal operator there is L which is represented by $\Box$ in this paper.

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