Notre Dame Journal of Formal Logic
Volume XIII, Number 3, July 1972
NDJFAM

A NOTE ON E

ALEKSANDAR KRON

Since there is no characteristic matrix for E so far, there is no possibility of investigating whether E has the finite model property in the sense of [1]. The aim of this note is to prove that for any wff D of E there is a finite set of wffs having properties similar to some properties of a finite model.

I shall suppose that E is formulated as in [2] or [3], but I shall write $\neg$ for negation instead of $\bar{}$. Let $X_1, X_2, \ldots$ be the sequence of all finite non-empty sets of wffs of E. If $X_i = \{A_1, \ldots, A_n\}$, $i = 1, 2, \ldots$, then $\overline{X}_i$ shall denote the wff $A_1 \,\&\, \ldots \,\&\, A_n$. Let us write X instead of X_i. X will be called *consistent* iff $\dashv_E \neg\overline{X}$; X is *inconsistent* iff $\vdash_E \neg\overline{X}$. Clearly, if X is consistent, then for no wff $B \vdash_E \overline{X} \rightarrow B \,\&\, \neg B$.

Lemma 1. *For any X, B and C, if X is consistent and $\vdash_E \overline{X} \rightarrow B \vee C$, then either $X \cup \{B\}$ or $X \cup \{C\}$ is consistent.*

Proof. Suppose that the contrary is the case. Then we have both $\vdash_E \neg(\overline{X} \,\&\, B)$ and $\vdash_E \neg(\overline{X} \,\&\, C)$. By adjunction we obtain $\vdash_E \neg(\overline{X} \,\&\, B) \,\&\, \neg(\overline{X} \,\&\, C)$ and thus $\vdash_E \neg(\overline{X} \,\&\, B \vee \overline{X} \,\&\, C)$. But then we easily derive $\vdash_E \neg(\overline{X} \,\&\, (B \vee C))$ and $\vdash_E \neg\overline{X} \vee \neg(B \vee C)$. Since $\vdash_E \overline{X} \rightarrow B \vee C$, we have $\vdash_E \neg(B \vee C) \rightarrow \neg\overline{X}$. Therefore, $\vdash_E \neg\overline{X}$, contrary to the assumption of the lemma.

Lemma 2. *For all X, B, C and D, if $\vdash_E \overline{X} \rightarrow B \vee C$ and $\dashv_E \overline{X} \rightarrow D$, then either $\dashv_E \overline{X} \,\&\, B \rightarrow D$ or $\dashv_E \overline{X} \,\&\, C \rightarrow D$.*

Proof. Suppose that both $\vdash_E \overline{X} \,\&\, B \rightarrow D$ and $\vdash_E \overline{X} \,\&\, C \rightarrow D$. We first easily obtain $\vdash_E (\overline{X} \,\&\, B) \vee (\overline{X} \,\&\, C) \rightarrow D$ and then $\vdash_E \overline{X} \,\&\, (B \vee C) \rightarrow D$. Since $\vdash_E \overline{X} \rightarrow B \vee C$, we have $\vdash_E \overline{X} \rightarrow \overline{X} \,\&\, (B \vee C)$ and thus $\vdash_E \overline{X} \rightarrow D$, contrary to the hypothesis of the lemma.

Let D be an arbitrary wff of E, let $\mathrm{P}^+(D)$ be the set of all subformulae of D, let $\mathrm{P}^-(D)$ be the set of all negations of the wffs of $\mathrm{P}^+(D)$ and let $\mathrm{P}(D) = \mathrm{P}^+(D) \cup \mathrm{P}^-(D)$. Furthermore, let $\mathrm{X}(D) = \{C_j \vee \neg C_j\colon\ C_j \in \mathrm{P}^+(D)\}$, for all $1 \leq j \leq r$, where r is the number of subformulae of D. In the sequel I shall consider only the members $Y_1, \ldots, Y_{2r}$ of the sequence $X_1, X_2, \ldots$ satisfying the following two conditions:

Received June 26, 1971