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AN EQUATIONAL AXIOMATIZATION AND A SEMI-LATTICE THEORETICAL CHARACTERIZATION OF MIXED ASSOCIATIVE NEWMAN ALGEBRAS

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In [2][1] and [3] M. H. A. Newman constructed and investigated two algebraic systems which he calls the fully complemented non-associative mixed algebra and the fully complemented associative mixed algebra respectively. In [6], [7], [8] and in this paper these systems are called simply: Newman algebras and associative Newman algebras. In [4] Newman constructed and investigated two relatively complemented algebraic systems which in some respect correspond respectively to the systems mentioned above. He calls, *cf.* [4], p. 38, these systems "mixed non-associative algebra" and "mixed (associative) algebra." In this paper only the latter system will be investigated and it will be called "mixed associative Newman algebra."

In [4], p. 40, the following characterization (the meaning of which will be explained in section 1 below)

Theorem 3 (Newman). *In order that a double algebra may be a mixed algebra it is necessary and sufficient that it be distributive and idempotent, that* $a(bb) = (ab)b$, *and that there exist a right* ω *and a left* ω.

of mixed associative Newman algebra has been established. Moreover, in [4], it has been proved that this algebraic system whose two basic binary operations are + and × is the direct join of an associative Boolean ring (without unity element) and a generalized Boolean algebra in the sense of Stone, *cf.* [9], p. 721, section 3.

In this paper it will be shown that, as in the case of the fully

[1] An acquaintance with the papers [2], [3], [4], [6], [7] and [8] is presupposed. In [2], [3] and [4] "*ab*" is used instead of "$a \times b$". An enumeration of the algebraic tables, *cf.* section 5 below, is a continuation of the enumeration of such tables given in [6] and [8].

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