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DUALS OF SMULLYAN TREES

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1. As readers of Jeffrey or Smullyan know, the consistency of a finite set S of wffs from the sentential calculus (SC) can be tested by means of a tree, called here a *Smullyan tree*.[1] The branches of the tree, which are gotten by breaking up each member of S into shorter wffs, breaking up these shorter wffs into still shorter ones, and so on, represent the various ways in which the members of S could be true. Those branches (if any) on which both an atomic wff (one of the letters 'P', 'Q', 'R', etc.) and its negation occur are said to be *closed*, the rest to be *open*. And the method guarantees that:

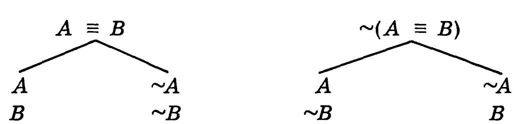
(1) If every branch of the tree is closed, S (the set tested) is inconsistent, whereas

(2) If at least one branch stays open, S is consistent, and a truth-value assignment on which all the members of S are true can be read off any open branch of the tree.

When '$\sim$', '&', 'v', and '$\supset$' serve as primitive connectives, the rules for breaking up truth-functional compounds are seven in number:[2]

1. Concerning Smullyan trees, see [4], [5], and [6]. We of course take a set S of the sort described to be (semantically) consistent if there is a truth-value assignment to the atomic components of the members of S on which all these members are true (i.e., get a **T**).

2. When '$\equiv$' also serves as a primitive connective, two extra rules serve to break up compounds of the sort $A \equiv B$ or the sort $\sim(A \equiv B)$:



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