

THE EXISTENCE POSTULATE AND NON-REGULAR  
 SYSTEMS OF MODAL LOGIC

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The reader is advised to read section 6 [8, pp. 178-198] before proceeding with this paper, and have the book in hand. If we refer to a certain page number without indicating its source it always refers to [8]. In this paper we shall use such terms as existence, proposition, etc. rather loosely. This is done on purpose to establish the continuity between Lewis's motivations and these investigations. In future we expect to give precise definitions of these terms and present further results about the systems described here. See also [6, pp. 290-292]. What follows has a close connection with certain remarks made there although those remarks are made in terms of models and "worlds".

1. *Preliminaries.* As we all know the systems of strict implication of Lewis are put forward as rivals to the system of material implication and constructed with the specific purpose of removing the "paradoxes" of the latter system. One such paradox is the following thesis:

$P1 \quad ACpqCpNq$

As an immediate consequence we have:

$P2 \quad NKNCpqNCpNq$

Consequently, "if we take ' $p$  is consistent with  $q$ ' to mean ' $p$  does not imply the falsity of  $q$ ' and ' $q$  is independent of  $p$ ' to mean ' $p$  does not imply  $q$ ', then in terms of material implication, no two propositions can be at once consistent and independent [p. 122]." This violates our intuitions; in other words, is paradoxical. Lewis thus constructs his systems so that in none of them the strict analogue of  $P1$ :

$P3 \quad A\mathcal{C}pq\mathcal{C}pNq$

is provable. Having constructed his systems Lewis noticed, however, that although  $P3$  was not a theorem of any of his systems it could be added with impunity to each of them, i.e., no inconsistency results on its addition.

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