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AN UNSOLVABLE PROVABILITY PROBLEM FOR ONE VARIABLE GROUPOID EQUATIONS

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The question for finite sets of equations suggested in the above title* is related to earlier work by the author [2] but was, in fact, suggested to him by Trevor Evans.

We deal with a relational or word calculus for semigroup presentations on two letters, say a and b, and concurrently with an equational or term calculus in one binary operation symbol and a single variable x. The letters a and b are *words*, and if W is a word, so are Wa and Wb. The variable x is a *term* and if s and t are terms, so is $(s + t)$. We use the natural notion of subterm, the viewpoint of terms as trees, and the assumption that some convenient system of ordering occurrences or locations of subterms within each term has been given.

The rules for deduction in both calculi are essentially the same except that in the equational case we are allowed to substitute a term for a variable uniformly throughout an equation. More precisely, let r, s, t, u denote terms in the variable x and C, D, F, G, W, V words in a and b. Let $t(r: x)$ stand for the term obtained by substituting r for all occurrences of x in t, and let $t[r: u: n]$ stand for the term obtained by using r to replace u at its location n in t if such exists and for t itself otherwise. The following deductions are allowed.

Equational Calculus	Relational Calculus
E1 $s = s$ from the empty set	**R1** $F = F$ from the empty set
E2 $s = t$ from $t = s$	**R2** $F = G$ from $G = F$
E3 $s = t$ from $s = r$ and $r = t$	**R3** $F = G$ from $F = W$ and $W = G$
E4 $s(r: x) = t(r: x)$ from $s = t$	**R4** $FW = GW$ from $F = G$
E0 $s = s[r: u: k]$ from $r = u$	**R5** $WF = WG$ from $F = G$

Let E be a set of equations. A finite sequence of equations $e_1, e_2, \ldots, e_n$ is a *proof of* e_n *from* E if each e_i is either an element of E or it is

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