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PUTTING K IN ITS PLACE

GEORGE F. SCHUMM

1 The *Dugundji axioms*[1]

$$A_n \qquad \bigvee_{1 \leq i \neq j \leq 2^{n+1}} \Box (p_i \leftrightarrow p_j) \qquad (n \in \omega)$$

are of particular interest in the study of extensions of S4 having the finite model property, since for each n the nth axiom has the attractive feature of being validated by a reflexive transitive generated frame $\mathfrak{F}$ if and only if $\mathfrak{F}$ contains at most n worlds. Where $\mathbf{L}$ is any extension of S4 and $\mathbf{L}_n$ is the smallest normal extension of $\mathbf{L}$ to contain A_n, it is natural to make the following

Conjecture *If* $\mathbf{L}$ *is determined by any class of frames, then* $\mathbf{L}_n$ *is determined by the class of frames for* $\mathbf{L}$ *which contain at most n worlds.*

Some partial results along these lines were announced in [6] where I first entertained this conjecture. But thanks to the recent work of Fine [2] something even stronger can now be established. Since A_n implies

$$\bigwedge_{1 \leq i \leq 2^{n+1}} \Diamond p_i \rightarrow \bigvee_{1 \leq i \neq j \leq 2^{n+1}} \Diamond (p_i \wedge (p_j \vee \Diamond p_j))$$

in the field of S4, it follows from Fine's very general completeness theorem that any extension of S4 having the axiom as a theorem is determined by the class of its at-most-n-membered frames. This has as an unexpected corollary that there are extensions $\mathbf{L}$ of S4 for which

$$\mathbf{L} \neq \bigcap_{n \in \omega} \mathbf{L}_n.$$

For as Fine has shown elsewhere [3], there exist extensions of S4—indeed a continuum of them—which are determined by *no* class of frames.

1. Made famous by Dugundji [1] and later used by Scroggs [8] to axiomatize all consistent extensions of S5. The axioms, as well as Dugundji's results, were evidently familiar to McKinsey at least a year before the appearance of Dugundji's paper (*cf.* [5]).

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