MORE LOGICS WITHOUT TAUTOLOGIES

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"Logic without tautologies"[*1] describes a system of sentential natural deduction (there called **C**) as "next-strongest": the addition to **C** of any general rule of inference yields a system which is either inconsistent or is a system of natural deduction for classical sentential calculus. **C** is not uniquely next-strongest. Another (non-equivalent) system, call it C_1, is also next-strongest. **C** and C_1 and other systems related to them are all of them logics without tautologies, and are all completable by the addition to them of the law of excluded middle in the form $S_1 \vdash S_2 \vee \sim S_2$.

1 *Replacement rules*   To discuss these matters, the results and conventions of notation and abridgement of proofs and the like of "Logic without tautologies" are herewith assumed, but with some simplifications. Below are repeated the primitive replacement rules of **C**, which are common to all systems under discussion.

C10. $\sim(S_1 \cdot S_2) \leftrightarrow \sim S_1 \vee \sim S_2$
C11. $S_1 \vee S_2 \leftrightarrow S_2 \vee S_1$
C12. $S_1 \vee (S_2 \vee S_3) \leftrightarrow (S_1 \vee S_2) \vee S_3$
C13. $S_1 \cdot (S_2 \vee S_3) \leftrightarrow (S_1 \cdot S_2) \vee (S_1 \cdot S_3)$
C14. $S \leftrightarrow \sim \sim S$
C16. $S_1 \supset S_2 \leftrightarrow \sim S_1 \vee S_2$
C17. $S_1 \equiv S_2 \leftrightarrow (S_1 \supset S_2) \cdot (S_2 \supset S_1)$
C17′. $S_1 \equiv S_2 \leftrightarrow (S_1 \cdot S_2) \vee (\sim S_1 \cdot \sim S_2)$
C19. $S \leftrightarrow S \vee S$

"Logic without tautologies" contains suggestions for deriving the remaining replacement rules—duals of **C**10-13, 19, and versions of exportation-importation and contraposition—of Copi's system from these, and a proof that either of **C**17, **C**17′ is derivable from the other in **C**. To see that neither of **C**17, **C**17′ is derivable from the remaining primitive