Notre Dame Journal of Formal Logic
Volume XIX, Number 3, July 1978
NDJFAM

TRUTH IN CONSTRUCTIVE METAMATHEMATICS

JOHN STAPLES

1 *Introduction* Metamathematics may be divided into two parts: that which relies on the use of logic, and that which does not. Most metamathematics which uses logic uses classical (two-valued) logic: we shall call it *classical* metamathematics. It is of course possible to consider metamathematics which uses logic, but which intends for the logic a constructive interpretation; we shall call that *constructive* metamathematics. There is a need for greater interest in constructive metamathematics, for example, to deal with theories which are in some way not standard, but which are claimed to admit a constructive interpretation. This paper is (apart from some asides) intended as a contribution to constructive metamathematics.

2 *Tarski's notion of truth*

2.1 In classical metamathematics the word "truth" has been given a technical meaning by Tarski, apparently without causing too much confusion with whatever is one's primary, intuitive notion of the meaning of the word. Tarski's notion of truth is not however confined to classical metamathematics. Its role is to describe the meaning of the logical concepts in a theory, in terms of the logical notions underlying the metatheory; it can do that in constructive metamathematics just as well as in classical metamathematics. When one considers the fundamental role of Tarski's notion in classical metamathematics, it is of some interest to know whether it can have any comparable role in constructive metamathematics.

Consider Tarski's definition of truth. It defines the truth of compound formulas and open formulas of a first order theory in terms of the truth of the closed atomic formulas, as follows. The definition is recursive, and it is assumed that the variables of the theory are intended to range over a fixed set S of objects; also that each relation symbol is intended to refer to a specific relation on that set of objects. We can also assume that the theory has a constant corresponding to each object in S (if it does not, extend the theory by adding such constants, define truth for the extended theory and take truth for the original theory to be the restriction of that

Received April 7, 1976