Notre Dame Journal of Formal Logic
Volume XIX, Number 3, July 1978
NDJFAM

S4.1.4 = S4.1.2 and S4.021 = S4.04

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In [2], modal systems S4.1.4 and S4.021 have been introduced as the result of restricting the proper axioms of S4.4 and S4.04, i.e.,

R1 $p \supset (MLp \supset Lp)$
L1 $p \supset (LMLp \supset Lp)$,

to

R1.3 $(p \supset Lp) \supset (ML(p \supset Lp) \supset L(p \supset Lp))$
L1.3 $(p \supset Lp) \supset (LML(p \supset Lp) \supset L(p \supset Lp))$

respectively. Since **R1.3** could be proven to be logically weaker than **R1**, the author thought it "very probable that **L1.3** [would similarly] not entail **L1**" ([2], p. 162). Also, since S4.021 could be proven to contain the strongest proper subsystem of S4.04, viz. S4.02, *properly*, the author thought (though rather diffidently) that S4.1.4 might likewise contain the strongest proper subsystem of S4.4, viz. S4.1.2, properly. The aim of this note is to disprove these two assumptions.

As chance would have it, the former assumption which seemed to be the more likely one turned out to be somewhat easier to refute than the latter. The following rather straight-forward derivation shows that, even in the field of S2, **L1.3** entails **L1**:

(1) $(\neg p \supset L\neg p) \supset (LML(\neg p \supset L\neg p) \supset L(\neg p \supset L\neg p))$	**L1.3**, $p/\neg p$
(2) $p \supset (\neg p \supset L\neg p)$	**PC**
(3) $LMLp \supset LML(\neg p \supset L\neg p)$	S2°
(4) $p \supset (LMLp \supset L(\neg p \supset L\neg p))$	(1)-(3)
(5) $L(\neg p \supset L\neg p) \supset L(Mp \supset p)$	S1°
(6) $L(Mp \supset p) \supset (LMp \supset Lp)$	S2°
(7) $LMLp \supset LMp$	S2
L1 $p \supset (LMLp \supset Lp)$	(4)-(7)

Hence S4.021 = {S4; **L1.3**} $\rightleftarrows$ {S4; **L1**} = S4.04.

With respect to **R1.3**, we obtain in an analogous way:

Received February 6, 1978