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# SEMANTICS FOR S4.1.2

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Sobociński's modal system S4.1 is obtained [8] by adding

**N1** *LCLCLCpLppCMLpp*

to a Gödel-style base for S4, and Zeman's S4.04 can be gotten (see [1]) by using

**L2** *CpLCMLpp*

instead. If both additions are made together, the system S4.1.2 of Sobociński's [9] results. Semantics for the former systems are available in [7] and [1], respectively; the aim of the present note is to provide them for S4.1.2 as well. Familiarity with modal semantics and Henkin-style completeness proofs in the approximate manner of [4] is presupposed.

Lemma 1 *The theorems of* S4.1.2 *are valid in each model* $\langle W,R,V\rangle$ *wherein* $R$ *is reflexive, transitive and satisfies*

$$\forall x\forall y\forall z\forall z'((xRy\,.\,yRz\,.\,xRz')\rightarrow(z'Ry\vee z=y\vee y=x))\qquad\text{(a)}$$

*Proof*: Since, as is well-known, reflexivity and transitivity ensure validation of S4's axioms, and detachment and necessitation preserve validity in any case, it is sufficient to show that neither **L2** nor **N1** can fail in models of the sort specified in the Lemma. And for **L2** we have only to note that identification of $z$ and $z'$ in (a) delivers, for reflexive, transitive models,

$$\forall x\forall y\forall z((xRy\,.\,yRz)\rightarrow(zRy\vee y=x)),\qquad\text{(b)}$$

a version of Goldblatt's S4.04 condition ([1], p. 393).

So suppose **N1** fails in some model $\langle W,R,V\rangle$ of the above sort. Then for some $x\in W$, $V(LCLCpLpp,x)=V(MLp,x)=\mathbf{T}$ but $V(p,x)=\mathbf{F}$. Since $V(LCpLp,x)=\mathbf{F}$, then, we must have $y$ in $W$ such that $xRy$, $V(p,y)=\mathbf{T}$ and $V(Lp,y)=\mathbf{F}$. The latter requires existence of at least one $z$ in $W$ for which $yRz$ and $V(p,z)=\mathbf{F}$; and since we had, earlier, $V(MLp,x)=\mathbf{T}$, we may also find $z'$ in $W$ such that $xRz'$ and $V(Lp,z')=\mathbf{T}$. Were $R$ as in the statement of the Lemma, then since $xRy$, $yRz$ and $xRz'$ we should have to have,

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