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PASSAGES BETWEEN FINITE AND INFINITE

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Infinite sets and infinite operations (e.g., infinite sums and products) arise from the generalization and extension of the concepts of finite sets and finite operations. However, due to the unintuitive nature of the concept of infinity, there is a natural tendency to handle problems involving infinite sets or infinite operations by means of finite sets or finite operations. In this way, problems involving finite sets or finite operations are passed on to problems involving infinite sets or infinite operations and vice-versa. It is by means of these passages between finite and infinite that problems involving the concept of infinity are handled in Mathematics.

In a given context, the passage between infinite and finite is predominantly performed by associating with an infinite set S a unique finite set $L(S)$ which may be called *the label* of S. In a more general situation, $L(S)$ need not be restricted to being finite. However, in this paper, to dramatize the passage from infinite to finite, we restrict $L(S)$ to being finite. For example, in the context of Mathematical Analysis, the infinite set $\{1/2, 1/3, 1/4, \ldots\}$ is very often associated with the finite set $\{0\}$. Similarly, the infinite set $\{3/4, 9/4, 4/5, 11/5, 5/6, 13/6, \ldots\}$ is very often associated with the finite set $\{1, 2\}$. The labels given depend on the context of the problem. Thus, in another context, a set other than $\{0\}$ may be associated with the infinite set $\{1/2, 1/3, 1/4, \ldots\}$. Depending on the purpose and the context, the labeling of infinite sets by means of finite sets can acquire various degrees of complexity. In some cases, extremely difficult situations may arise when the lebeling of sets must satisfy certain properties or fulfill certain requirements.

On the other hand, if the labeling of infinite sets by finite sets is not subject to any specific conditions, then the labeling can be done quite simply, say as a function assigning a finite set to every infinite (or for the sake of generality to every) set of the universe of discourse. Even with this degree of arbitrariness, the passage between infinite and finite is achieved with the utmost theoretic rigor. Clearly, the labeling can be considered as a table with two columns, one listing the sets and the other listing their labels.