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PROOFS OF THE NORMALIZATION AND CHURCH-ROSSER THEOREMS FOR THE TYPED λ-CALCULUS

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Introduction This paper contains new proofs of the normalization theorem and Church-Rosser theorem for the typed λ-calculus. Both results are obtained as corollaries of a theorem which shows that a certain kind of reduction sequence must always contain a normal term.[1] The proof of this theorem proceeds *via* an assignment of ordinals. A knowledge of ordinal arithmetic sufficient for understanding this assignment[2] will be presupposed, and detailed arguments for various assertions about alphabetic change of bound variables will not be given. Apart from these matters the paper is self-contained.

1 *The calculus* Terms are built up from variables $x, y, z, x_1, \ldots$, the operator λ, and the grouping indicators) and (according to the following rules.[3]

1. x is a *term*.
2. If t and u are terms, then (tu) is a *term*.
3. If t is a term, then (λxt) is a *term*.

Henceforth $t, u, v, t_1, \ldots$ are to be terms. Omitted parentheses are to be restored according to the convention of association to the left, and a dot is to be construed as a left parenthesis which has its mate as far to the right as possible. The formulas of the propositional calculus which can be built up from propositional parameters $p, q, r, s, p_1, \ldots$, the connective $\supset$, and the grouping indicators will be used as *type symbols*. In what follows $A, B, C, A_1, \ldots$ are to be type symbols. '$\equiv$' will be used to express identity. τ_0 is to be a function which maps the set of variables onto the set

1. It is *not* shown that every reduction sequence must contain a normal term.

2. Rubin [1, pp. 175-219] is enough.

3. The use/mention conventions of Curry will be employed—all symbols written down are in the metalanguage and the objectlanguage is never displayed.

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