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CLASSIFICATIONS FOR INCONSISTENT THEORIES

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In [2] N. C. A. da Costa surveys some interesting results about inconsistent formal systems. A formal system is said to be inconsistent if there is a formula φ such that both φ and $\sim\varphi$ are theorems. The approach in [2] towards the study of inconsistent systems is basically syntactical. In this paper we investigate inconsistent theories from a model-theoretical point of view. However we do not analyze semantically the calculi presented in [2] as suggested on Page 508. Instead we define a notion of structure which allows for the possibility of built-in inconsistencies. These structures may then be models of inconsistent theories. We classify theories in 3 different ways. Intuitively, the higher a theory is in a classification, the more inconsistent it is. This way we obtain measures of inconsistency for theories.

1 *Terminology and Examples* Since for the purposes of this paper it is convenient to deviate somewhat from the standard terminology, we explain our notations in this section. We deal with first-order languages of finite type with equality and without function symbols. A type $\mu = \langle n_1, \ldots, n_k\rangle$ is always finite and nonempty. We use j, k, m, n for integers or possibly ω; α, β for infinite cardinals; φ, ψ for formulas (usually sentences); Γ for a set of sentences. The cardinality of a set A is denoted by $|A|$. We differentiate between equations and atomic formulas: an equation has the form $t_i = t_j$ while an atomic formula has the form $S_i(t_1, \ldots, t_n)$ where S_i is an n_i-ary relation symbol mentioned in μ, and the t_i are terms. We use the connectives $\sim$, $\wedge$, $\vee$, and the quantifiers $\exists$, $\forall$.

We give the following recursive definition of a formula:

1) Every equation, negation of equation, atomic formula, and negation of atomic formula is a formula.
2) If φ and ψ are formulas then so are $\varphi\wedge\psi$, $\varphi\vee\psi$, $\sim\varphi(\exists x)\varphi$, and $(\forall x)\varphi$.
3) An expression is a formula only if it follows from a finite number of applications of 1) and 2) that it is a formula.

Sometimes we may write an expression where negation is applied to a formula which is neither an equation nor an atomic formula. Such an

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