

A NOTE CONCERNING A SOLE SUFFICIENT OPERATOR

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Let $E(k) = \{0, 1, \dots, k - 1\}$. In [3] Wesselkamper proves that a particular Markov operator S defined on $E(k)$ is complete with constants (a function is complete with constants if the set consisting of the function and all the constant functions is complete). In this short note we give an alternative proof of the completeness by showing that Rose's generalized condition disjunction [2] is easily defined using S and the constant functions. This conditioned disjunction is defined by $[x, y_0, \dots, y_{k-1}] = y_i$ if $x = i$ and it was proved in [2] to be complete with constants. (Rose repeats the variable x at the end of the definition to maintain symmetry.)

$$S \text{ is defined by } Sxyz = \begin{cases} z, & \text{if } x = y \\ x, & \text{otherwise.} \end{cases}$$

We use three sets of definitions, as follows:

$$\begin{aligned} \text{D1.} \quad & V_0x = SOSx001 \\ & V_jx = SOSjx01, \text{ for each } j = 1, 2, \dots, k - 1. \end{aligned}$$

$$\text{Then } V_ix = \begin{cases} 1, & \text{if } x = i; \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{D2.} \quad W_ixy = SV_ix1y, \text{ for each } i, 0 \leq i \leq k - 1.$$

$$\text{Hence } W_ixy = \begin{cases} y, & \text{if } x = i, \\ 0, & \text{otherwise.} \end{cases}$$

$$\begin{aligned} \text{D3.} \quad & Q_1xy_0y_1 = SW_0xy_0W_1xy_1 \\ & Q_{i+1}xy_0y_1 \dots y_{i+1} = SQ_ixy_0 \dots y_i 0W_{i+1}xy_{i+1} \text{ for each } i = 1, 2, \dots, k - 2. \end{aligned}$$

Then $[x, y_0, \dots, y_{k-1}] = Q_{k-1}xy_0 \dots y_{k-1}$ so that S is complete with constants.