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HYPERENUMERATION REDUCIBILITY

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There are several ways of reducing a set B to a set A and the differences are mostly related to the basic procedures allowed in the reduction (recursive, arithmetical, etc.) but also to the manner in which the input set A enters in the reduction. In some cases both inputs from A and the complement $\overline{A}$ are allowed in the reduction (as in Turing reducibility) and in other cases the reduction operates only on positive information about A (as in enumeration reducibility). It is important to see that in some cases the positive reduction is actually a generalization of the positive-negative reduction with the same basic procedures. For instance the Turing reduction $B \leq_T A$ can be in fact defined as $C_B \leq_e C_A$ where C_A is (the graph of) the characteristic function of A. This does not mean that the structures induced by the reductions (the so-called degrees) are similar. In fact it is well known there are important differences between the ordering of Turing degrees and partial degrees.

In this paper we present a form, a positive reduction, which we call hyperenumeration reducibility. It is related to hyperarithmetical reduction exactly as enumeration reducibility is related to Turing reducibility. The basic procedures in hyperenumeration reducibility are analytical involving function quantifiers. Since any form of positive reduction is essentially weak this strengthening of the basic procedures seems to be a desirable feature. We attempt, in this paper, a classification of sets of natural numbers in terms of hyperenumeration reducibility. The basic ideas have been first applied to enumeration reducibility and in Section **1** we give the fundamental definitions. The ideas and results in this section are introduced mainly as a motivation for the material in the remaining sections. Hyperenumeration reducibility is defined in Section **2**, and in Section **3** we introduce the fundamental notion of pseudo hyperarithmetical set. The structure of degrees containing such sets is discussed. Finally, we prove there are sets of degrees in which there is no pseudo hyperarithmetical set. The construction here follows a forcing technique which was first introduced by Thomason in [6]. We shall use the notation of Rogers [5].

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