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AWKWARD AXIOM-SYSTEMS

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As far as I know a notion of the awkward axiom-systems, which is introduced in this note, was never previously discussed in the appropriate literature in which the theory of the well-formed axiomatizations is investigated. I define the notion mentioned above as follows:

Let T be an axiomatizable theory with the rules of procedure $\mathcal{R}$. Let $\mathfrak{X}$ be a finite or infinite axiom-system of T based on the rules $\mathcal{R}$ which satisfies the following conditions:

(1) The axioms belonging to $\mathfrak{X}$ are mutually independent.
(2) The set of the axioms belonging to $\mathfrak{X}$ can be divided into two subsets Γ and Δ having no common elements and such that

$$\mathfrak{X} = \bigcup\{\Gamma, \Delta\}$$

where Γ is a finite set, and Δ can be either finite or infinite.
(3) There is a finite set θ of formulas which are the consequences of the set of axioms Γ obtained from Γ by the applications of the rules $\mathcal{R}$, but without the use of the axioms belonging to Δ and such

(a) that θ is a proper subsystem of Γ,

and

(b) that the theses belonging to the set $\bigcup\{\theta, \Delta\}$ are mutually independent.

Then, if on the base of the rules $\mathcal{R}$ system $\bigcup\{\theta, \Delta\}$ is inferentially equivalent to $\bigcup\{\Gamma, \Delta\}$, the axiom-system $\mathfrak{X}$ is awkward.

Clearly, although from the points of view of the other requirements concerning the well-formed axiomatizations, the awkward axiom-systems are entirely correct, it is self-evident that in such axiomatizations there are accepted some axioms (belonging to Γ) which are stronger than needed and, on the other hand, the deductive power of the other axioms (belonging to Δ) is not used. Hence, the awkward axiom-systems can be considered as not especially elegant axiomatizations. There are several theories, mostly

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