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ON CREATIVE DEFINITIONS IN FIRST ORDER FUNCTIONAL CALCULI

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Russell's widely accepted claim that definitions are "theoretically superfluous" is partially vindicated by the

Theorem *In the first order functional calculus definitions of the type*

$$\Theta\alpha_1\alpha_2 \ldots \alpha_n \equiv \omega$$

are not creative, i.e., no new theorems, in primitive notation, are provable using definitions. (See [3], p. 190, for proof).

In this note we point out that this theorem is imprecisely stated in that it is dependent upon the particular axiomatic presentation of the first order functional calculus which is chosen. We do this by giving an axiomatization with respect to which there is a definition of the above type which is creative. As axiom schemata take the following:

(1) $E \equiv F \supset . A \supset . B \supset A$
(2) $E \equiv F \supset . A \supset [B \supset C] \supset . A \supset B \supset . A \supset C$
(3) $E \equiv F \supset . \sim A \supset \sim B \supset . B \supset A$
(4) $E \equiv F \supset . (a)[A \supset B] \supset . A \supset (a)B$, *where a is any individual variable which is not a free variable of A.*
(5) $E \equiv F \supset . (a)A \supset \mathsf{S}_b^a A|$, *where a is an individual variable, b is an individual variable or an individual constant, and no free occurrence of a in A is in a well formed part of A of the form* $(b)C$.
(6) $A \equiv B \supset . A \supset B$
(7) $A \equiv B \supset . B \supset A$
(8) $\sim[A \supset B \supset \sim[B \supset A]] \supset . A \equiv B$

In this presentation $\supset$, $\sim$, and $\equiv$ are primitive.[1] The rules are detachment and generalization.

1. Axioms (6)-(8) are needed since $\equiv$, as well as $\sim$, $\supset$, is primitive. Without them no connection between $\equiv$ and $\sim$, $\supset$ would be provable. One could take $\sim$, $\supset$ as the only primitives. Then read $\supset$ for $\equiv$ in (1)-(5) and drop (6)-(8). But then the rule of definition must allow for the introduction of pairs of implications ($A \supset B$, $B \supset A$) rather than equivalences ($A \equiv B$).

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