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MINIMALIZATION OF BOOLEAN POLYNOMIALS, TRUTH FUNCTIONS, AND LATTICES

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1 *Introduction* In 1952 W. V. Quine [1] published a landmark paper on the minimalization of a Boolean polynomial or truth function. First, the polynomial is fully expanded out to the "developed form", and then one finds the "prime implicants" by employing a tabular representation to find common indicators. In 1956 E. J. McCluskey published a sequel to Quine's paper [2], in which a decimal coding scheme is employed on the binary-number equivalents of the terms of the expanded polynomial, to facilitate the simplification process. The Quine procedure, or Quine-McCluskey procedure as it is sometimes referred to (see Phister [3] or Korfhage [4]), is used in the logical design of digital computers and communications equipment, to help reduce the number of logical circuits required. The purpose of this paper is to develop a set-theoretic explanation for the process of minimalization of a Boolean polynomial, with an algorithm for minimalization which is simpler than that of Quine-McCluskey, in that a full expansion of the polynomial and a tabular representation is not necessary. Thus, the procedure is carried out algebraically, in a way which is amenable to processing entirely by a digital computer. The procedure also includes a test to determine whether a particular form is minimal.

A Boolean polynomial for a system with n binary variables represents a lattice within the universal set of 2^n binary n-tuple elements. Each term (frequently called "minterm" of this polynomial is a monomial which represents a "complete subset (sublattice)" of n-tuple elements identified by m common-valued binary variables, $m < n$, which are used as indicators for the subset.[1] The objective in minimalization is to reduce the size of the polynomial, as a description for the lattice, so that it has both the smallest number of terms and the smallest average number of indicators per term. This is achieved when you find the smallest number of complete sublattices which cover the entire function. This is essentially the same thing as

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