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Notre Dame Journal of Formal Logic
Volume XIX, Number 1, January 1978
NDJFAM

FORBIDDEN SUBGRAPHS IN TERMS OF FORBIDDEN QUANTIFIERS

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In 1930, Kuratowski characterized planar graphs as those graphs which fail to contain either of two special subgraphs; see Theorem 11.13 of [4]. Since then, such "forbidden subgraph" characterizations have been sought and prized by graph theorists. The nature of such characterizations is considered in [2] and [3]. In particular, [3] is based on the simple observation that a class of graphs has a forbidden subgraph characterization if and only if the class contains each subgraph of each of its members.

We will show that the properties characterizable using forbidden subgraphs are precisely those which are expressible in a natural symbolic language from which existential quantifiers have been forbidden. Of course, this is exactly what is expected from combining the observation of [3] with the well known result of Tarski and Łoś on properties preserved under subsystems (Theorem U of [5]). But unlike either of these approaches, ours uses only very simple symbolic logic and is actually able to produce the set of forbidden subgraphs.

While following the graph-theoretic terminology of [4], one important distinction must be stressed. We call H an *induced subgraph* of G if H results by removing points from G (along with each line incident with a removed point). On the other hand, H is a *subgraph* of G if H results by removing lines or points from G (along with each line incident with a removed point). (The notion of containment in Kuratowski's theorem is slightly different from each of these.)

Consider the language $\mathcal{L}$ involving variables $x, y, \ldots$ (interpreted as points) and the binary relations $=$, $\neq$, $\sim$, and $\nsim$ (interpreted as equality, nonequality, adjacency, and nonadjacency). Also, $\mathcal{L}$ has the connectives $\wedge$ and $\vee$ (for conjunction and disjunction) and universal and existential quantifiers. The universal $\mathcal{L}$-sentences are defined in the expected manner. Note that the omission of a symbol for negation in no way limits the expressiveness of $\mathcal{L}$, since occurrences of negation can be reduced to uses of $\neq$ and $\nsim$.

Received June 22, 1977