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A NOTE ON THE LAW OF IDENTITY AND THE CONVERSE PARRY PROPERTY

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On a few occasions Anderson and Belnap in [2] are eager to stress the importance of the law of identity for E. In this note we shall give some results bearing upon the role of the law of identity in the implicational and implication-negation fragment of E. Our notation and basic conceptual apparatus will be the same as in [2]. Moreover we define the following.

An entailment subformula (**ef**) of a wff A of $\mathbf{E}_{\rightarrow}$ or $\mathbf{E}_{\overset{\rightarrow}{\neg}}$ is any subformula of A of the form $B \rightarrow C$. An elementary **ef** (**eef**) of A is any **ef** which has only a propositional variable on at least one side of the arrow (e.g., $D \rightarrow p$, $p \rightarrow D$, and $p \rightarrow q$ are all **eef**s). A minimal **ef** (**mef**) of A is any **eff** of the form $p \rightarrow q$.

We can now state:

Lemma 1 *If every* **ef** *B of $\vdash_{E_{\rightarrow}} A$ is $\vdash_{E_{\rightarrow}} B$, then*

1.1. *every* **eef** *of A is of the form $p \rightarrow p$,*
1.2. *A contains only one propositional variable.*

Proof: 1.1. Every **mef** will be of the form $p \rightarrow p$ in virtue of variable-sharing. **Eef**s of the form $p \rightarrow C$ and $C \rightarrow p$, where C is an **ef**, are ruled out, the first because of the Ackermann property, the second because by *modus ponens* p would be a theorem. So every **eef** is a **mef**, and Lemma 1.1. follows.

1.2. Let A contain two or more propositional variables. In virtue of variable-sharing every **ef** of A containing two or more propositional variables will have on at least one side of its arrow a subformula containing at least two propositional variables (so this subformula will be an **ef**). (E.g., imagine an **ef** B containing two propositional variables, p and q; then it will contain p either on both sides of the arrow, in which case q is on at least one side, or on only one side, in which case q must be on both.)*

*Occasionally, in virtue of the converse Parry property (v. infra) we know that the **ef** on the right-hand side of the arrow will always contain two or more propositional variables. Of course this fact is not essential for the proof.

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