Notre Dame Journal of Formal Logic
Volume XIX, Number 1, January 1978
NDJFAM

THE INTENSIONALITY OF THE PREDICATE '___ IS RECURSIVE'

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G. Lee Bowie's "An Argument Against Church's Thesis" provides an opportunity to clarify some points about our concepts of recursive and computable. In [1], G. Lee Bowie argued that there is a noncomputable recursive function and that there are computable functions which are very likely nonrecursive. Against his first claim, I shall argue, primarily, that Bowie has instead exposed the intensional character of the predicate '___ is recursive.' Against his second claim, I shall argue that he has used an eccentric sense of 'computable.'

Bowie does not want to identify functions with rules of correspondence or descriptions of correspondences so he uses Church's λ-operator. So, before beginning my rebuttal, I will show how I will use this operator as well as settle some terminological points. A set S is a *non-negative integer correspondence*, or alternatively: a sequence, if S is a set of ordered pairs of non-negative integers $\langle x, y\rangle$ such that for every x there is exactly one y such that $\langle x, y\rangle$ is in S. Let us call non-negative integer correspondences simply 'correspondences,' and call these correspondences 'functions.' We can here identify functions with such correspondences because we will restrict our attention to function descriptions of one argument place whose domain is the non-negative integers and whose range is a subset of the non-negative integers. By regarding functions as sets such as these correspondences we follow usual mathematical practice and Church in section 03 of [2] where he writes on p. 16: "In other words, we take the word 'function' to mean what may otherwise be called a *function in extension*." However, some authors such as Church himself in [3] and Yasuhara in [4] do not identify functions with extensions such as correspondences; they identify functions with rules for generating correspondences. When he identified functions with rules, Church wrote on p. 3 of [3]: "we shall say we are dealing with *functions in intension*." Although we will not deal with functions in intension, we will talk of the intensions of functions.

The λ-operator provides notation for presenting an intension of a

Received September 5, 1973