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A FORMAL SYSTEM FOR THE NON-THEOREMS OF THE PROPOSITIONAL CALCULUS

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Introduction The completeness of the classical propositional calculus allows us to give a deductive system consisting of finitely many *axiom schemas* and finitely many *rules of inference*, that permit us to pass from a formula or a pair of formulae to a syntactically related formula, in such a manner that the formulae obtained inductively from the axioms by repeated application of the rules are exactly the tautologies. In this paper we give an analogous deductive system (more concretely, a Hilbert type system) such that the formulae deduced are exactly those that *are not* tautologies, the non-theorems of the propositional calculus. Obviously, this has to be the most non-standard of the non-classical logics. It is important to note that there are many other algorithms to generate recursively the non-theorems, since the propositional calculus is decidable. Usually they are based in the methodical search for a counterexample, but they lack the inductive character of a Hilbert type system, where every formula involved in a deduction is itself deductible. In our system, unlike semantic tableaux or refutation trees, every formula introduced in a deduction is a non-tautology, and it is introduced only if it is a non-tautological axiom, or it follows by one of the non-tautological rules of inference from non-tautologies introduced earlier in the deduction.

1 *Axioms and rules* We assume that the only connectives are $\sim$ and $\supset$. $p, q, p_1, p_2, \ldots$ denote atomic formulae. $\alpha, \beta, \gamma, \ldots$ denote arbitrary formulae. We define $\mathcal{P}(\alpha) = \{p | p \text{ occurs in } \alpha\}$.

Axioms

A1 $p \supset \sim p$ (p atomic)
A2 $\sim p \supset p$ (p atomic)

Rules

R1 (a) $\dfrac{\alpha}{p \supset \alpha}$ (p atomic, p does not occur in α)

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