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A SHORTEST SINGLE AXIOM FOR THE CLASSICAL EQUIVALENTIAL CALCULUS

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Ten shortest single axioms for the classical equivalential calculus **E** are known. Of these three are due to Łukasiewicz [2] and the remaining seven to Meredith [5], p. 185. Meredith gave proofs for two of his axioms, and the other five were proved by Peterson [8], who found his proofs with the aid of the theorem-proving computer program described in [9]. Meredith also stated (*cf.* [5], p. 185 and [10], p. 307) that

(6) *EpEEqErpEqr*

(we follow the numbering of [8]) is a single axiom for **E**, but this is incorrect, as noted by Peterson [8], p. 270.

The main object of this paper is to show that

(6′) *EpEEqErpErq*

is a single axiom for **E**; (6′) evidently corrects a misprint in (6) in both [5] and [10]. The proof that (6′) is a single axiom for **E** was found with the aid of a computer program similar to Peterson's, and an account of the way in which the computer obtained this proof is included below. Further discussions of the use of computers to prove theorems in Hilbert-type sentential calculi may be found in [1] and [6].

The following deduction shows that

(10) *EEEpEqrrEqp*

is derivable from (6′) by condensed detachment; for the proof that (10) is a single axiom for **E**, see [8], p. 270.

1.	*EpEEqErpErq*	
2.	*EEpEqErEEsEtrEtsEqp*	= **D**1.1
3.	*EEEEpEqrEqpErss*	= **D**2.1
4.	*Epp*	= **D**3.3
5.	*EEpEqErrEqp*	= **D**1.4
6.	*EEpEpqq*	= **D**5.1

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