92
Notre Dame Journal of Formal Logic
Volume XIX, Number 1, January 1978
NDJFAM

FOUNDATIONAL PROBLEMS OF NUMBER THEORY

YVON GAUTHIER

I want to address myself in this paper* to the thesis that realism is a viable philosophy of mathematics and I am going to attack the thesis by examining number theory and by proposing a foundational framework that purports to explain fundamental results in arithmetic and analysis without having to resort to realism. My main philosophical target or pretext will be Hilary Putnam's philosophy of mathematics (the pre-Marxist one).

1 Putnam is said to have argued that ontological realism can be combined with an empiricist epistemological attitude which would assimilate mathematics to theoretical physics at least as far as hypothetico-deductive reasoning and inductive confirmation of hypotheses are concerned. For example, it would be legitimate according to Putnam, to accept the truth of Fermat's last theorem and Goldbach's conjecture on the ground that these hypotheses have been tested sufficiently to warrant our acceptance—Fermat's last theorem asserts that it is impossible to find a natural number with a power greater than two such that it is the sum of two other natural numbers with the same power, e.g., a cube cannot be the sum of two cubes, etc. . . . and Goldbach's conjecture says that every even number from six onwards can be represented as the sum of two prime numbers other than two. Now if one is interested in number theory, one knows that in the case of Goldbach's conjecture related problems have been solved e.g., "Every sufficiently large odd number is representable as the sum of three odd primes" and "Almost every even number is representable as the sum of two odd primes."[1]

But I want to go a little further: an important theorem by Dirichlet says that if a and b are relatively prime (such that they have no common divisor, 1 excepted), there are infinitely many primes of the form $ax + b$, that is in any arithmetical progression. Dirichlet gave a proof of that statement using analytical methods (L-functions or Lolomorphic functions

*I am indebted to Bas C. van Fraassen, Gonzalo Reyes and Georg Kreisel for helpful comments.

Received June 10, 1974