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# NECESSITAS CONSEQUENTIS IN A SINGLETON POSSIBLE WORLD

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In [1], M. J. Cresswell adopts a Kripke semantics according to which "a one-world model is a model in which $Lp$ and $p$ have the same truth-value and in which $CpLp$ is true." In [5], I proved that not only is the system $M'(T')$, with $CpLp$ as a thesis, formally consistent, but it does not collapse into classical sentential calculus. Now I wish to show that there is a sense of "possible world," closely allied to that of Kripke, in which $Lp$ and $p$ do not necessarily have the same truth-value and in which $CpLp$ is contingent.

In Kripke [3], $R$ is idle in a normal model structure $\langle G, K, R\rangle$ where $K = \{G\}$. That is, $R$ fails to distinguish between Kripke [3] and Kripke [2]. Now, in Kripke [2], a possible world is a truth-value assignment to every atomic subformula of a wff $\alpha$. We depart from Kripke in this—that, for us, a possible world is not a truth-value assignment to atomic variables. It is a set of such assignments. Following Massey [4], we understand by a plenary set $\Omega$ a set of partial and complete truth-tables for a wff $\alpha$ such that any truth-value assignment $\Sigma$ to the variables of $\alpha$ is represented in some member of $\Omega$.

We let a member of $\Omega$ represent a possible world. That is, we let a partial or complete truth-table for a wff $\alpha$ represent a set of truth-value assignments for a wff $\alpha$. The semantics for '$L$' are then stipulated, not across possible worlds but within them as in Massey [4].

$L_1$

| $\alpha$ | $L\alpha$ |
|---|---|
| t | t |

$L_2$

| $\alpha$ | $L\alpha$ |
|---|---|
| f | f |

$L_3$

| $\alpha$ | $L\alpha$ |
|---|---|
| t | f |
| f | f |

Now, consider the following plenary set of truth-tables for $CpLp$.

$T_1$

| $P$ | $CpLp$ |
|---|---|
| t | t t |

$T_2$

| $P$ | $CpLp$ |
|---|---|
| f | t f |

$T_3$

| $P$ | $CpLp$ |
|---|---|
| t | f f |
| f | t f |

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