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A SEMANTICAL ANALYSIS OF THE CALCULI $\mathbf{C}_n$

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1 *Introduction* $\mathbf{C}_1$ is a propositional calculus which can serve as a basis for inconsistent, but non trivial deductive theories (see [1] and [2]). The axiomatic basis of $\mathbf{C}_1$ is as follows:

1. Primitive symbols of $\mathbf{C}_1$: 1.1. $\supset$ (implication), & (conjunction), $\vee$ (disjunction), and $\neg$ (negation); 1.2. propositional variables: $p, q, r, \ldots, p', q', r', \ldots$; 1.3. Parentheses.

The notion of formula and the symbol of equivalence ($\equiv$) are defined in the standard way. Roman capitals will be used as syntactical variables for formulas. A° is an abbreviation of $\neg(A \,\&\, \neg A)$.

Definition 1 $\neg^* A =_{df} \neg A \,\&\, A^\circ$.

2. Postulates (axiom schemata and deduction rule) of $\mathbf{C}_1$:

(1) $A \supset (B \supset A)$,
(2) $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$,
(3) $\dfrac{A \quad A \supset B}{B}$,
(4) $A \,\&\, B \supset A$,
(5) $A \,\&\, B \supset B$,
(6) $A \supset (B \supset A \,\&\, B)$,
(7) $A \supset A \vee B$,
(8) $B \supset A \vee B$,
(9) $(A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C))$,
(10) $A \vee \neg A$,
(11) $\neg\neg A \supset A$,
(12) $B^\circ \supset ((A \supset B) \supset ((A \supset \neg B) \supset \neg A))$,
(13) $A^\circ \,\&\, B^\circ \supset (A \,\&\, B)^\circ$,
(14) $A^\circ \,\&\, B^\circ \supset (A \vee B)^\circ$,
(15) $A^\circ \,\&\, B^\circ \supset (A \supset B)^\circ$.

(Formal) proof, deduction and the symbol $\vdash$ are introduced as in Kleene's book [4].

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