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AN AXIOM SYSTEM FOR THREE-VALUED ŁUKASIEWICZ PROPOSITIONAL CALCULUS

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0 *Introduction* In 1920, Łukasiewicz has introduced the notion of three-valued logic. It was not constructed as a formalized axiomatic deductive system but was built up by means of the truth-table method. The matrix defining this logic is the following [3], p. 166:

C	0	$\frac{1}{2}$	1	N
0	1	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$
1	0	$\frac{1}{2}$	1	0

The three-valued Łukasiewicz logic was later axiomatised by Wajsberg in 1931 (see [3], p. 291). Moisil has given an axiomatisation in order to show that the three-valued Łukasiewicz propositional calculus is an extension of the intuitionistic one. We give here another axiomatisation, different from that of Moisil, showing that the three-valued Łukasiewicz propositional calculus is an extension of a fragment of the three-valued intuitionistic propositional calculus (see [1]; [3], p. 286), answering a problem suggested by A. Monteiro.

Łukasiewicz characteristic matrix can be considered as an algebraic structure. In 1940, Moisil has introduced the notion of three-valued Łukasiewicz algebra as an attempt to give an algebraic approach to the three-valued propositional calculus considered by Łukasiewicz. Following Monteiro [6], we can define a three-valued Łukasiewicz algebra in the following way, where the primitive operations are those chosen by Moisil. Thus an abstract algebra $\langle A, \wedge, \vee, \sim, \nabla, 1\rangle$ is said to be a three-valued Łukasiewicz algebra provided that $\langle A, \wedge, \vee, 1\rangle$ is a distributive lattice where 1, $\sim$, and ∇ are two unary operations on A such that

$$\begin{aligned} \sim\sim x &= x \\ \sim(x \wedge y) &= \sim x \vee \sim y \\ \sim x \vee \nabla x &= 1 \\ x \wedge \sim x &= \sim x \wedge \nabla x \\ \nabla(x \wedge y) &= \nabla x \wedge \nabla y \end{aligned}$$

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