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# ON RAMSEY'S THEOREM AND THE AXIOM OF CHOICE

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It is known that Ramsey's theorem cannot be proved in **ZF** without the axiom of choice (see, e.g., Kleinberg [2]) but there does not seem to exist in the literature, or at least be widely recognized, a clear cut statement of the exact relationship between this combinatorial result and the principle of choice (in Drake [1], p. 72, the problem is mentioned but only a partial answer is given). The aim of this note* is to write down a proof of the

Proposition *Ramsey's theorem is equivalent to the axiom of choice for countable families of finite sets.*

For a set $X$, let $[X]^2$ be the set of unordered pairs from $X$; if $f\colon [X]^2 \to 2$ is a partition of $[X]^2$ into two disjoint sets, a set $Y \subseteq X$ is said to be homogeneous for $f$ if $f \restriction [Y]^2$ is constant. Then by Ramsey's theorem we mean the statement

**(RT)** *Any partition* $f\colon [X]^2 \to 2$ *of an infinite set* $X$ *possesses an infinite homogeneous set*

which is the crucial step of Ramsey [3].

We abbreviate with **(CCF)** the axiom of choice for countable families of non-empty finite sets; **(CCF)** is equivalent in **ZF** to König's lemma

**(KL)** *Any infinite finitary tree has an infinite branch*

and also **(KL)** $\Rightarrow$ **(RT)** (see, e.g., Drake [1], p. 203). It remains to be shown that **(RT)** $\Rightarrow$ **(KL)**; we prove it in a roundabout way through the following weak form of compactness for propositional logic

**(CPL)** *Let S be a countable set of propositional sentences over an infinite set of propositional letters; then S has a model iff every finite subset of S has a model.*

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