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MODAL TREE CONSTRUCTIONS

RALPH L. SLAGHT

1 The utility of truth tree constructions to determine the validity of truth-functional and/or quantificational arguments is well-known. In what follows, I have extended the procedure of Jeffrey[1] for the purpose of also handling arguments whose sentences contain the standard modal operators, $\Box$ and $\Diamond$. The basic program has been designed to accommodate modal system T (Hughes and Cresswell),[2] including first-order logic with the Barcan formula, $(x)\,\Box(\ldots x \ldots) \supset \Box(x)(\ldots x \ldots)$. The two modal notions which are essential to the modal tree constructions for T are: (a) if $\Diamond p$ is a sentence (see section **2.1** below) in tree A, then p is a sentence in some alternative-world tree to A; and (b) if $\Box p$ is a sentence in A, then p is a sentence in every alternative-world tree which has access to A (see section **2.4** below).

2 *Definitions and Notes*

2.1 "Sentence" in these contexts is elliptical for "either a sentence or a sentence-form" and it refers to a point in a tree, not to components of sentences which make up the point.

2.2 "Constructed configuration" means "All the sentences, trees, paths, and alternative-world trees which have been written down as a result of a particular application of the Program for modal tree constructions".

2.3 A *path* is a sequence of points in a tree such that the origin of the tree is in every path which is in the tree and such that every point below the origin is a successor of some previous point.

2.4 A path, B, *has access to* an alternative-world tree, V, just in case both

1. Richard C. Jeffrey, *Formal Logic: Its Scope and Limits*, McGraw-Hill, New York (1967).

2. G. E. Hughes and M. J. Cresswell, *An Introduction to Modal Logic*, M. J. Cresswell, London (1968), pp. 22-42. System T was originally propounded by Robert Feys in "Les logiques nouvelles des modalités," *Revue Néoscolastique de Philosophie*, vol. 40 (1937), pp. 517-553 and vol. 41 (1938), pp. 217-252.

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