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THE ENTAILMENT OPERATOR

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Finding an adequate definition of the entailment operator is a fundamental concern of logicians. Such an explication would, hopefully, reveal that entailment, long assumed to be a logical relationship, can satisfactorily be treated in some truth-tabular way. This paper attempts to follow out a few heartfelt and deeply entrenched assumptions of logicians on the basis of some recently advanced hypotheses concerning the nature of entailment. It is also one exemplification of the treatment of this relation by means of a many-valued, modal, truth-tabular system of logic. The application of these techniques to this set of assumptions and hypotheses should yield a plenary set of tables which will be paradigmatic and, thus, definitive of the entailment operator.

1 *Starting Points* Let us make the following assumptions:

(1) Entailment is a logical relation; that is, one that is not essentially dependent upon factors which cannot be formalized.
(2) Entailment is a relation that obtains between statements.
(3) Entailment is, as such, amenable to truth-tabular treatment.
(4) A relation is amenable to truth-tabular treatment if, and only if, it can be expressed as a logical operator such that its function is capable of being characterized by a set of matrices.
(5) Every statement has a logically manipulatable "value", ("true", "false", "0", "1", "2", etc.) which can perform as either an argument or a value in a logical function.

Let us operate with the following hypotheses:

(6) Entailment is not adequately expressed by any two-valued modal or non-modal truth-tabular set of matrices.
(7) An adequate analysis of the entailment relationship is:

A statement S entails a statement E if, and only if, S's being true is a

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