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AN AXIOMATIZATION OF HERZBERGER'S 2-DIMENSIONAL PRESUPPOSITIONAL SEMANTICS

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The purpose of this paper* is to axiomatize two 4-valued propositional logics suggested by Herzberger in [1], section VI. They are of philosophical interest because their interpretation makes use of two ideas inspired by Jean Buridan: (1) a proposition may correspond to the world and yet be untrue because it is semantically deviant, and (2) logically valid arguments preserve correspondence with reality, not truth. If the two non-classical truth-values of these systems are identified, the resulting tables for the classical connectives are the weak and strong systems of Kleene. Unlike Kleene's system, the 4-valued ones offer a choice of designated values that renders semantic entailment perfectly classical. Compare Herzberger [2] and Martin [5].

Let the set $\mathcal{F}$ of formulas be inductively defined over a denumerable set of atomic formulas such that $\neg A$, $A \& B$, $\mathsf{C}A$, $\mathsf{B}A$, $\mathsf{T}A$, $\mathsf{F}A$, $\mathsf{t}A$, and $\mathsf{f}A$ are formulas if A and B are. Let $\mathcal{W}$ be the set of all $\mathfrak{w}$ such that for some $\boldsymbol{v}$ and $\mathfrak{v}$,

(1) for any atomic formula A, $\boldsymbol{v}(A)$, $\mathfrak{v}(A) \in \{0, 1\}$;
(2) $\boldsymbol{v}(\neg A) = 1$ if $\boldsymbol{v}(A) = 0$; $\boldsymbol{v}(\neg A) = 0$ otherwise;
$\boldsymbol{v}(A \& B) = 1$ if $\boldsymbol{v}(A) = \boldsymbol{v}(B) = 1$; $\boldsymbol{v}(A \& B) = 0$ otherwise;
$\boldsymbol{v}(\mathsf{C}A) = 1$ if $\boldsymbol{v}(A) = 1$; $\boldsymbol{v}(\mathsf{C}A) = 0$ otherwise;
$\boldsymbol{v}(\mathsf{B}A) = 1$ if $\mathfrak{v}(A) = 1$; $\boldsymbol{v}(\mathsf{B}A) = 0$ otherwise;
$\boldsymbol{v}(\mathsf{T}A) = 1$ if $\boldsymbol{v}(A) = v(A) = 1$; $\boldsymbol{v}(\mathsf{T}A) = 0$ otherwise;
$\boldsymbol{v}(\mathsf{F}A) = 1$ if $\boldsymbol{v}(A) = 0$ and $\mathfrak{v}(A) = 1$; $\boldsymbol{v}(\mathsf{F}A) = 0$ otherwise;
$\boldsymbol{v}(\mathsf{t}A) = 1$ if $\boldsymbol{v}(A) = 1$ and $\mathfrak{v}(A) = 0$; $\boldsymbol{v}(\mathsf{t}A) = 0$ otherwise;
$\boldsymbol{v}(\mathsf{f}A) = 1$ if $\boldsymbol{v}(A) = v(A) = 0$; $\boldsymbol{v}(\mathsf{f}A) = 0$ otherwise;
(3) $\mathfrak{v}(\neg A) = 1$ if $\mathfrak{v}(A) = 1$; $\mathfrak{v}(\neg A) = 0$ otherwise;
$\mathfrak{v}(A \& B) = 1$ if $\mathfrak{v}(A) = \mathfrak{v}(B) = 1$; $\mathfrak{v}(A \& B) = 0$ otherwise;
$\mathfrak{v}(\mathsf{C}A) = \mathfrak{v}(\mathsf{B}A) = \mathfrak{v}(\mathsf{T}A) = \mathfrak{v}(\mathsf{F}A) = \mathfrak{v}(\mathsf{t}A) = \mathfrak{v}(\mathsf{f}A) = 1$;

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