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BINARY CONSISTENT CHOICE ON TRIPLES

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1 *Introduction* Łoś and Ryll-Nardzewski introduced various principles of "consistent" choice with respect to symmetrical relations in [4], [5] and then showed many were equivalent to **P.I.**, the prime ideal theorem for Boolean algebras.[1] In particular, they showed that even for binary relations, consistent choice from finite sets of cardinality $\leqslant n$ equals **P.I.**, for $n = 4, 5, 6, \ldots$. Here we extend this result to include $n = 3$.

2 Let A be a collection of sets and R a binary symmetric relation. A set t is a *choice set* for A if $t \bar{\bar{\cap}} a = 1$, for all $a \in A$; if, in addition, $\{x, y\} \in R$ for all x, y in t with $x \neq y$, t is an *R-consistent* choice set for A. The collection of all choice sets for A will be denoted by $\mathrm{c}(A)$, while the collection of all R-consistent choice sets is $\mathrm{c}_R(A)$. In [4], [5], the following theorem was proved equivalent to **P.I.**

Theorem 1 *Let A be a collection of finite sets and R a binary symmetric relation, and suppose that for any finite $A_0 \subset A$, $\mathrm{c}_R(A_0) \neq \emptyset$. Then $\mathrm{c}_R(A) \neq \emptyset$.*

Let F_n denote the statement of Theorem 1 if the sets of A are further restricted to have cardinality $\leqslant n$; then, as mentioned above, Łoś and Ryll-Nardzewski even showed $F_n \leftrightarrow$ **P.I.**, $n = 4, 5, 6, \ldots$. We will prove $F_3 \leftrightarrow$ **P.I.** It is, of course, enough to show $F_3 \rightarrow$ **P.I.**

Let $\beta = \langle B, \wedge, \vee, \sim, 0, 1\rangle$ be a Boolean algebra. For any $K \subset B$, let $\tilde{K} = \{\{b, \sim b\} | b \in K\}$. If $K \subset B$ is a subalgebra, any prime ideal of K is an element of $\mathrm{c}(\tilde{K})$. Moreover, any ideal of K which belongs to $\mathrm{c}(\tilde{K})$ is a prime ideal of K. Let $\mathrm{pr}(K)$ denote the set of prime ideals of K and let $\Sigma(B) = \{K \subset B | K$ is a finite subalgebra of $\beta\}$. It is easy to see that any $I \in \mathrm{c}(\tilde{B})$ will be a prime ideal of β if $I \cap K$ is an ideal of K, for all $K \in \Sigma(B)$.

Theorem 2 $F_3 \rightarrow$ **P.I.**

Proof: Let $\beta = \langle B, \wedge, \vee, \sim, 0, 1\rangle$ be a Boolean algebra. For each finite

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1. Equivalent here means in **ZF** without the axiom of choice.

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