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THE ONE-ONE EQUIVALENCE OF SOME GENERAL COMBINATORIAL DECISION PROBLEMS

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1 *Introduction* A *general combinatorial decision problem* may be defined quite simply to be a family of related decision problems concerned with some class of combinatorial systems. E.g., the general halting problem for Turing machines is the family of halting problems ranging over all Turing machines. Let G_1 and G_2 be two general combinatorial decision problems. G_1 is said to be *one-one* (*many-one*) *reducible* to G_2 if there exists an effective mapping ψ from the problems p in G_1 into the problems $\psi(p)$ in G_2 such that p is of the same one-one (many-one) degree as $\psi(p)$. (Actually if p is solvable we only require that $\psi(p)$ be also solvable.) G_1 and G_2 are said to be *one-one* (*many-one*) *equivalent* if each is one-one (many-one) reducible to the other. Recent research by the authors and Overbeek [2, 3, 4, 5, 6, 7, and 10] has demonstrated the many-one equivalence of a large number of general combinatorial decision problems. In this paper* we will show that some of these general decision problems are in fact one-one equivalent. Our method of proof, which has been used by Cleave [1] to study "system functions", is to show that each non-recursive instance of the general decision problems under consideration is a cylinder. Since many-one equivalence of cylinders implies one-one equivalence, the desired results are achieved.

2 *Cylinders and their properties* Let R be an arbitrary recursively enumerable (r.e.) set. R is called a *cylinder* if the decision problem for membership in R is of the same one-one degree as that for the set of pairs $\{\langle x, n\rangle \mid x \in R$ and n is a natural number$\}$. That is to say, R is a cylinder if it may be placed in an effective one-one correspondence with the cartesian

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