Notre Dame Journal of Formal Logic
Volume XVIII, Number 2, April 1977
NDJFAM

ON YABLONSKII THEOREM CONCERNING FUNCTIONALLY COMPLETENESS OF k-VALUED LOGIC

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In his paper, S. B. Yablonskii [1] proved a theorem concerning the functional completeness in k-valued logic (see [1], p. 64). The theorem asserts that the system of functions consisting of constant $k - 2$, $\sim x$, and $x_1 \supset x_2$ is functionally complete in this logic. His proof is incomplete. In this paper, we shall give a simple proof of this theorem.

Let P_k be the set of all functions that are defined on the set $\{0, 1, \ldots, k - 1\}$ and take their values on the same set. First, we shall give a lemma needed for the proof of the theorem.

Lemma *The system consisting of functions* $0, 1, \ldots, k - 1$, $\max(x_1, x_2)$, $\min(x_1, x_2)$ *and* $j_i(x) (0 \leq i \leq k - 1)$ *defined by*

$$j_i(x) = \begin{cases} k - 1, & \text{if } x = i, \\ 0, & \text{if } x \neq i, \end{cases}$$

is functionally complete in P_k.

Proof: We use the induction. All the constants are already given. If we put

$$\max(y_1, y_2, \ldots, y_n) = \max[\max\{\ldots \max(\max(y_1, y_2), y_3) \ldots\}, y_n],$$

then

$$f(x_1, \ldots, x_n, x_{n+1}) = \max[\min\{f(x_1, \ldots, x_n, 0), j_0(x_{n+1})\},$$
$$\min\{f(x_1, \ldots, x_n, 1), j_1(x_{n+1})\}, \ldots, \min\{f(x_1, \ldots, x_n, k - 1), j_{k-1}(x_{n+1})\}].$$

Therefore, from the induction hypothesis we can construct every $n + 1$-variable function in P_k by superposition. The lemma is proved.

Now we shall prove the following theorem:

Theorem *The system of functions consisting of the constant* $k - 2$, $\sim x$, *and* $x_1 \supset x_2$, *where* $x_1 \supset x_2 = \min(k - 1, x_2 - x_1 + k - 1)$, *is functionally complete in* P_k.

Received December 1, 1975