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THE MODAL PREDICATE LOGICS PF*F

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1 *Introduction* This paper introduces the five modal predicate logics **PF*F** that correspond to the five modal propositional logics **F*F** (**F** = **Ł**, **W**, **S**, **D**, **E**) of [1]. The notation used here follows Smullyan ([2] Chs. IV, V) except where indicated and Wilson [1], and hence many details are omitted. In section **2** the semantics **PF*** are discussed with emphasis on the semantics of quantified expressions. In section **3** the formal systems **PF** are given based again on Smullyan's 'analytic tableaux' [2]. In section **4** the semantical consistency and completeness proofs are illustrated with specific reference to **PS*S**. In the final section, section **5** further points and problems are mentioned, the key one being the possible bearing of these logics **PF*F** on 'traditional predication theory', at least when set against the back-cloth of Angelelli's observations in [3]. Throughout this paper, except where indicated, the index i ranges over 1, 2, 3, 4 and the index j ranges over 1, 2. (Similarly for **i** and **j**).

2 *The Semantics* PF*

2.1 For the syntax of **PF**, we add to the syntax of **F** (omitting propositional variables and the functor 'F'), the symbols '$\forall$' and '$\exists$', and denumerable lists of individual variables $x, y, z, \ldots$; individual parameters, $a, b, c, \ldots$ (the set Π); and for each positive integer n, n-ary predicates $P, Q, R, \ldots$ (in all cases with or without subscripts). We now give some definitions. An atomic formula of **PF** is defined as an $(n + 1)$-tuple $Pv_1 v_2 \ldots v_n$ where P is any n-ary predicate and v_i, $i = 1, 2, \ldots, n$, are any individuals (i.e., variables or parameters).

We can then define a wff in **PF** by making use of the formation rules for **F**, see [1], together with the new rule: If A is a wff in **PF** and x is a variable then $\forall xA$ and $\exists xA$ are wff. The definition of wff in **PF** can be made explicit in the usual way. Signed wff (swff) in **PF** are analogous to swff in **F**. We now define a closed wff (cwff) in **PF** as follows:

A is a cwff of **PF** if A is a wff of **PF** and for every variable x and every parameter a, $A^x_a = A$.

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